PHYSICAL REVIEW
LETTERS

VOLUME 5 AUGUST 15, 1960 NUMBER 4

SELF-DIFFUSION COEFFICIENT AND NUCLEAR SUSCEPTIBILITY OF LIQUID He^3†

A. C. Anderson,* H. R. Hart, Jr.,‡ and J. C. Wheatley‖
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 27, 1960)

The self-diffusion of the magnetization has been measured at pressures ranging from 5 to 9 cm-Hg in pure liquid He^3 (He^4 impurity less than $2:10^5$) and in liquid He^3 containing 1.2% He^4 from 0.15°K to 0.032°K.[1] The nuclear susceptibility relative to its value at the He^4 bath temperature has been measured at similar pressures in pure He^3 from 1.3°K to 0.031°K, and in He^3 containing 1.2% He^4 at 1.3°K and at a few low temperatures, the lowest being 0.032°K. No evidence has been found for a transition of the liquid into some sort of cooperative state.[2]

The diffusion coefficient, D, was measured using a spin echo method at a frequency of 85 kc/sec. A 90°-180°-180° pulse sequence was used with a fixed time delay between the 90° pulse and the first 180° pulse and a variable time, τ, between the two echoes. The resulting echoes were displayed on an oscilloscope and photographed. The ratio, R, of the amplitudes of the two echoes is $R=\exp(-\tau/T_2-\gamma^2G^2D\tau^3/12)$, where G is the magnetic field gradient, $\gamma=2.038\times10^4$ (gauss-sec)$^{-1}$, and T_2 is the transverse relaxation time.[3] The diffusion coefficient was calculated neglecting the effect of T_2. It was found that T_2 was greater than 0.74 sec at $T=0.034$°K. This magnitude for T_2 would lead to a +2.2% error in D at the lowest temperatures. The cell containing the He^3 was a cylinder 0.5 cm i.d. × 0.8 cm long. The copper cooling wires were embedded in the cylindrical surface and did not penetrate into the space occupied by He^3; hence, it is believed that the measured D is characteristic of the bulk liquid. Measurements of D were made as a function of G and with G alternately parallel and antiparallel to the steady field. Convection effects were assumed to be absent. The apparatus outside the cryostat and the method used to obtain the echoes were essentially the same as described by Hart and Wheatley.[4]

The relative nuclear susceptibility, χ_r, was measured using the apparatus described above but with no applied gradient and with one 180° pulse applied just a few milliseconds after the 90° pulse. The amplitude of the echo was taken to be proportional to the susceptibility provided the magnetizing time was long enough to allow the thermal equilibrium magnetization to develop. All echo amplitudes were divided by those measured at the He^4 bath temperature at the end of a day's measurements to obtain a quantity proportional to χ_r. As the temperature decreased, the amplitude became dependent on the time after the 90° pulse. The temperature-dependent gradient responsible for this defect in the experiment probably came from a weak magnetism of the "Epibond 104"[5] used to construct the He^3 cell. The time dependence was extrapolated out assuming that it arose from diffusion. However, the resulting values of χ_r, which have been increased by the extrapolation from the measured amplitudes by about 4% at 0.03°K, may still be as much as 7% too small at 0.03°K since it is not clear empirically how the extrapolation should be made. The correction was less than 1% for $T>0.065$°K.

Temperatures were measured using a slurry of cerium magnesium nitrate and Dow-Corning 200 Fluid of 50 cs viscosity encased in a shell of Epibond 104. At the conclusion of the meas-

urements of D and χ_γ it was shown that Epibond 104 was weakly magnetic. Since this material had been used rather extensively in construction, it was necessary to make very careful magnetic temperature calibrations of all parts of the apparatus. In the 1-4°K region, 27% of the Curie constant of the thermometer came from Epibond 104. A special experiment was made to determine the T-T^* curve for Epibond 104 using cerium magnesium nitrate as thermometer.[6] The presence of the Epibond constitutes a serious defect in the precision of the experiment; however, the measured temperatures below 0.15°K are less susceptible to error than those at higher temperatures where a thermal shield containing Epibond varied in temperature in a relatively unknown way.

The results of the diffusion measurements are shown in Fig. 1 and those for the relative susceptibility measurements are shown in Fig. 2. These results may be summarized as follows: (*i*) Adding 1.2% He^4 to pure He^3 does not change either D or χ_γ within our experimental error. (*ii*) Between 0.1°K and 0.03°K, D obeys a power law $(1/T)^n$, where $n = 1.55 \pm 0.05$ for pure He^3 and $n = 1.56 \pm 0.05$ for He^3 with 1.2% He^4. In reference 4, it was found that $n = 1.46 \pm 0.1$ in a more restricted temperature range with a 1.1% He^4 impurity and with a He^3 cell in which diffusion could have been limited somewhat by the boundaries. (*iii*) Between 0.1°K and 0.04°K, χ_γ is constant within an experimental error of about 3%. At 0.03°K, χ_γ appears to be a few percent lower, but this effect is thought to be spurious. (*iv*) Deviations at higher temperatures of χ_γ from constancy occur in the same temperature range as deviations of D from a strict power-law dependence on temperature. (*v*) The ratio of the susceptibility at 1.293°K to that below 0.1°K is 0.268 ± 0.008. If He^3 obeyed Curie's law at all temperatures, the susceptibility below 0.1°K would correspond to a temperature of (0.347 ± 0.010)°K. (*vi*) No evidence has been found down to temperatures of 0.03°K either in D or in χ_γ for a transition to some sort of cooperative state.[2]

Smoothed data from reference 4 are shown on Fig. 1. At low temperatures the curves are nearly parallel but shifted by about 14% from one another. This shift probably arises from a systematic error in calibration of the thermometer in the earlier experiment due to neglect of the weakly magnetic properties of some of the materials used in construction. The measure-

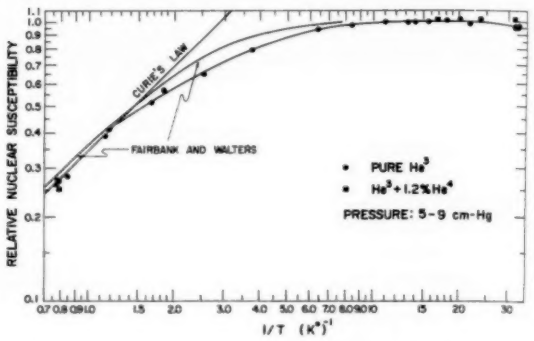

FIG. 2. Plot of the logarithm of the relative nuclear susceptibility against the logarithm of the reciprocal of the temperature for He^3 containing less than 2 parts in 10^5 of He^4 and for He^3 containing 1.2% He^4. The relative nuclear susceptibility is the ratio of the susceptibility at temperature T to that below 0.1°K. The curve labeled "Fairbank and Walters" represents smoothed data from reference 10. The straight line labeled "Curie's Law" is the curve the relative susceptibility would follow if He^3 obeyed Curie's law. The values of relative susceptibility have been corrected for changes in the density of He^3 with temperature.

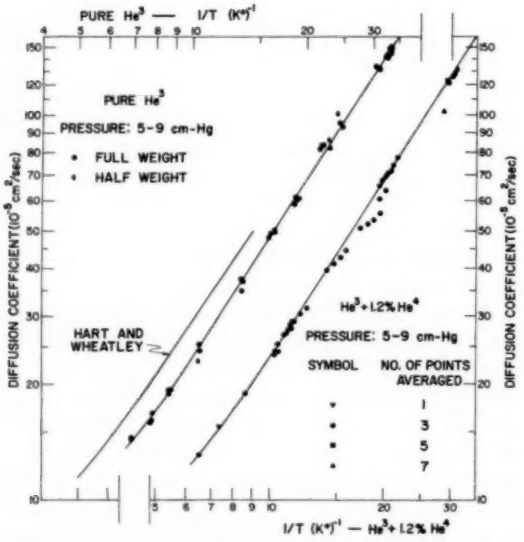

FIG. 1. Plot of the logarithm of the diffusion coefficient against the logarithm of the reciprocal of the temperature for He^3 containing less than 2 parts in 10^5 of He^4 and for He^3 containing 1.2% He^4.[1] The curve labeled "Hart and Wheatley" represents smoothed data from reference 4. The curve drawn through the data for He^3 with 1.2% He^4 is the one having maximum slope.

ment of the temperature dependence in the present experiment is marred by the necessity of accounting for the deviations of Epibond 104 from Curie's law. In the present experiment the ferric alum refrigerator, the thermometer, and the He^3 cell were completely different from those in reference 4. Since the same temperature dependence of D was observed in both experiments, within experimental error, it seems unlikely that serious systematic errors in the temperature dependence are present.

The straight line drawn through the experimental values of D for the He^3 containing 1.2% He^4 has the maximum reasonable slope. Experimental points to the right of this line were obtained on days when the warmup rate was much larger than normal, with the exception of an unexplainable point at $1/T = 28.7°K^{-1}$, $D = 102 \times 10^{-5}$ cm^2/sec. Hence the effect of the heat leak is to decrease the value of n in the law $D \propto (1/T)^n$. Similar effects were not observed in the measurements with pure He^3. Approximately three hours after demagnetization D reached a constant value characteristic of 0.03°K.

The ratio of the susceptibility at the He^4 bath temperature (corrected for variations in this temperature and for density[7]) to that below 0.1°K had a maximum spread of 3% for three measuring days and was independent of the calibration of the magnetic thermometer. Combining the measured value of the specific heat[8] with the above ratio, one finds that the ratio $\chi T/C_v$, where C_v is the specific heat and χ is the susceptibility below 0.1°K, is 4.8 ± 0.4 times larger than it would be in the absence of spin-dependent interactions.[9]

Values of χ_γ for pressures near the saturated vapor pressure have been calculated from the graphs given by Fairbank and Walters[10] and plotted on Fig. 2, χ_γ arbitrarily being set equal to 1.0 at their lowest temperature, 0.13°K. If their curve is normalized to agree with ours at the bath temperature, it is found that their extrapolated χ_γ agrees with the value measured in this experiment to within 2%. This agreement is within the experimental error of both experiments, which differ from one another quite considerably in details. Between $(1/T) = 2\ (K°)^{-1}$ and $(1/T) = 6\ (K°)^{-1}$, again normalizing Fairbank and Walters' curve to agree with ours at the bath temperature, the present values of χ_γ are as much as 8% lower than those of Fairbank and Walters. However, in this temperature range the errors in thermometry due to the presence of Epibond 104 in a thermal shield are difficult to assess and may be as large as 5 to 10%. Hence the deviations from the data of Fairbank and Walters may not be significant.

We would like to thank Professor John Bardeen, Professor William Fairbank, Mr. Daniel Hone, and Professor C. P. Slichter for discussing this problem with us; Professor Peter Yankwich for helping us with the mass spectrometric analysis of the helium gas; Dr. H. A. Reich for some helpful remarks on the design of the He^3 refrigerator used in the experiments; and Mr. R. J. Sarwinski and Mr. F. A. Franz for assistance in performing the experiments.

†This work has been supported by the Alfred P. Sloan Foundation, the U. S. Atomic Energy Commission, and the Office of Naval Research.

*National Science Foundation Pre-Doctoral Fellow.

‡Present address: General Electric Research Laboratory, Schenectady, New York.

‖A. P. Sloan Fellow.

[1]The 1.2% He^4 impurity is in the gas. It is possible that in the temperature range of our experiments there is a phase separation which changes the concentration of He^4 in the He^3-rich phase. K. N. Zinov'eva and V. P. Peshkov, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 33 (1959) [translation: Soviet Phys.-JETP 10, 22 (1960)].

[2]L. N. Cooper, R. L. Mills, and A. M. Sessler, Phys. Rev. 114, 1377 (1959); K. A. Brueckner, T. Soda, D. W. Anderson, and P. Morel, Phys. Rev. 118, 1442 (1960); L. P. Pitaevskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1794 (1959) [translation: Soviet Phys.-JETP 10, 1267 (1960)]; V. J. Emery and A. M. Sessler, Phys. Rev. 119, 43 (1960).

[3]H. Y. Carr and E. M. Purcell, Phys. Rev. 94, 630 (1954); R. L. Garwin and H. A. Reich, Phys. Rev. 115, 1478 (1959).

[4]H. R. Hart, Jr. and J. C. Wheatley, Phys. Rev. Letters 4, 1 (1960).

[5]Furane Plastics, Inc., 4516 Brazil Street, Los Angeles 39, California.

[6]"Epibond 104" has a susceptibility per gram in the region from 1°K to 2°K which is 31% of that for cerium magnesium nitrate. In the region of 0.04°K, $T^* \sim 2T$. The magnetism seems to come from the Bentonite filling material and hence probably arises from some impurity. Complete details of the T-T^* experiment will be supplied on request.

[7]E. C. Kerr, Phys. Rev. 96, 551 (1954); D. M. Lee and H. A. Fairbank, Phys. Fluids 2, 528 (1959).

[8]D. F. Brewer, J. G. Daunt, and A. K. Sreedhar, Phys. Rev. 115, 836 (1959).

[9]L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 1058 (1956) [translation: Soviet Phys.-JETP 3, 920 (1957)]; M. J. Buckingham, Proceedings of the Symposium on Solid and Liquid He^3 (Ohio State University, Columbus, 1957), p. 52.

[10]W. M. Fairbank and G. K. Walters, Proceedings of the Symposium on Solid and Liquid He^3 (Ohio State University, Columbus, 1957), p. 211.

GENERALIZED BARDEEN-COOPER-SCHRIEFFER STATES AND ALIGNED ORBITAL ANGULAR MOMENTUM IN THE PROPOSED LOW-TEMPERATURE PHASE OF LIQUID He^3

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

and

P. Morel
French Embassy, New York, New York
(Received July 19, 1960)

It has recently been observed[1-3] that there are solutions of the Bardeen-Cooper-Schrieffer (BCS) theory[4] of Fermion systems with attractive forces which correspond to condensing the Fermion pairs into $l=1,2,3,\ldots(p,d,f,\ldots)$ states[5] rather than into the spherically symmetric s state of BCS theory. In particular, Brueckner et al.[2] first pointed out that the forces in liquid He^3 are favorable to a condensation into an $l=2$ state, at a temperature of the order of 0.05°K. It is not inconceivable that such states might also appear in metals.

Although the existence of such states is as yet only a theoretical possibility, their properties seem interesting enough to bear full investigation. We have carried out such an investigation within the limitations of the BCS type of theory, and we present here a number of new results.

As in BCS, we confine ourselves to states only of zero total momentum, for which it is adequate[5] to take as our Hamiltonian

$$\mathcal{H}=\sum_{\vec{k}}(\epsilon_{\vec{k}}-\epsilon_F)n_{\vec{k}}+\tfrac{1}{2}\sum_{\vec{k},\vec{k}'}V_{\vec{k}\vec{k}'}c_{\vec{k}}^{\dagger}c_{-\vec{k}}^{\dagger}c_{-\vec{k}'}c_{\vec{k}'}. \quad (1)$$

(We omit the spin index; for l odd, the $\vec{k}$ and $-\vec{k}$ spins are to be thought of as parallel, for l even antiparallel.) An essentially exact ground state of (1) is obtained by solving the BCS integral equation,

$$\tan\chi_{\vec{k}}\ \exp(i\psi_{\vec{k}})=\frac{1}{2(\epsilon_{\vec{k}}-\epsilon_F)}\sum_{\vec{k}'}(-V_{\vec{k}\vec{k}'})\ \sin\chi_{\vec{k}'}\ \exp(i\psi_{\vec{k}'}), \quad (2)$$

where

$$\sin\chi_{\vec{k}}\ \exp(i\psi_{\vec{k}})=\langle 0|c_{\vec{k}}^{*}c_{-\vec{k}}^{*}|0\rangle \quad (3)$$

corresponds to the quantity $[h_{\vec{k}}^{1/2}(1-h_{\vec{k}})^{1/2}]^{1/2}$ in the usual BCS formula, with the essential generalization of allowing complex coefficients in the product wave function.

Similarly to Bogolyubov,[6] we may observe that (2) should already sum the "ladder diagrams" which are necessary to renormalize out the repulsive cores of the interactions. This summation is most simply carried out by introducing a cutoff energy $\Delta \gg$ (kT and all gap energies) into (2), in which case (2) can be shown to become

$$\tan\chi_{\vec{k}}\ \exp(i\psi_{\vec{k}})=2^{-1}(\epsilon_{\vec{k}}-\epsilon_F)^{-1} \times \sum_{|\epsilon_{\vec{k}'}-\epsilon_F|<\Delta} K_{\vec{k}\vec{k}'}\ \sin\chi_{\vec{k}'}\ \exp(i\psi_{\vec{k}'}), \quad (4)$$

where

$$K_{\vec{k}\vec{k}'}=-V_{\vec{k}\vec{k}'}+\sum_{|\epsilon_{\vec{k}''}-\epsilon_F|>\Delta} V_{\vec{k}\vec{k}''}\frac{1}{2|\epsilon_{\vec{k}''}-\epsilon_F|}K_{\vec{k}''\vec{k}'} \quad (5)$$

is the linear integral equation solved by K, a quantity like the "reaction matrix" of scattering theory.

Our new results are:

I. For weak enough forces (valid in He^3), we may choose Δ small enough that $V_{\vec{k}\vec{k}'}$ or $K_{\vec{k}\vec{k}'}$ does not vary rapidly with the magnitude of $\vec{k}$ or $\vec{k}'$ within Δ. Then one may expand

$$K_{\vec{k}\vec{k}'}=\sum_{l=0}^{\infty}U_l P_l(\hat{k}\cdot\hat{k}'), \quad (6)$$

with U_l constant. This justifies rigorously using the "cutoff potential" of BCS for weak-coupling cases. We have also proved explicitly that even though U_l depends on Δ, no physical properties do. In references 2 and 3 it is essentially shown that U_2 is the first positive (attractive) term for He^3.

A simple way to relate U_l to the interatomic forces is to calculate T_c using (6) and compare this to the calculations of T_c in reference 3, for

example. Our theorem ensures that the two should give the same T_c; this determines U_l for a given Δ.

II. An exact ground state may be obtained only if only one U_l is nonvanishing. In that case we see that by group theory

$$\sum_{|\epsilon_{\vec{k}'}-\epsilon_F|<\Delta} K_{\vec{k}\vec{k}'} \sin\chi_{\vec{k}'} \exp(i\psi_{\vec{k}'}) = 2\sum_m \Delta_m Y_m^l(\hat{k}). \quad (7)$$

We have found explicit solutions only for the case of only one nonvanishing Δ_m, in which case the excitation energies become

$$E_{\vec{k}} = \{(\epsilon_{\vec{k}}-\epsilon_F)^2 + (\Delta_m)^2[P_m^l(\hat{k}\cdot\hat{n})]^2\}^{1/2}, \quad (8)$$

where $\hat{n}$ is a unit vector in some arbitrary direction in space. Numerical results for Δ_m's and total energies will be published elsewhere. It suffices to point out that Δ_m decreases with m, but that the total energy increases; $m=l$ is always the best state. It is interesting to observe that kT_c, Δ, and the total energy decrease exponentially with l for given U_l and weak forces.

We have not considered mixed cases with several nonvanishing Δ_m's, except to show that the pure m solutions are all local minima or maxima of the energy. Probably $m=l$ is the true minimum.

III. Even when all U_l's are nonvanishing, the above may be shown to be nearly exact. We have computed the coupling between different U_l's; this is small, occurs (when $m=l$) only with U_{l+2}, $U_{l+4}\cdots$, and converges rapidly.

IV. The density in energy of excited states $n(E)$ behaves as $E^{2/l}$ when $E\to 0$, and goes to infinity at $E=\Delta_l$. Figure 1 is a sketch of $n(E)$. Note how as $l\to\infty$ the density approaches the normal case.

V. The thermodynamics is obtained by straightforward techniques based on BCS; explicit results will be published elsewhere. The only striking difference is implied by Fig. 1: The specific heat as $T\to 0$ varies as $T^{1+2/l}$, since there is not a true energy gap. The equation for T_c is linear and so T_c is independent of m for a given l.

VI. The most unusual properties are connected with the angular momentum. In BCS, one remembers, there is a correlation "hump" of the order of ϵ_0/ϵ_F electrons about any given electron, with a range $\sim\xi_0=(2/\pi k_F)(\epsilon_F/\epsilon_0)$. This is shown

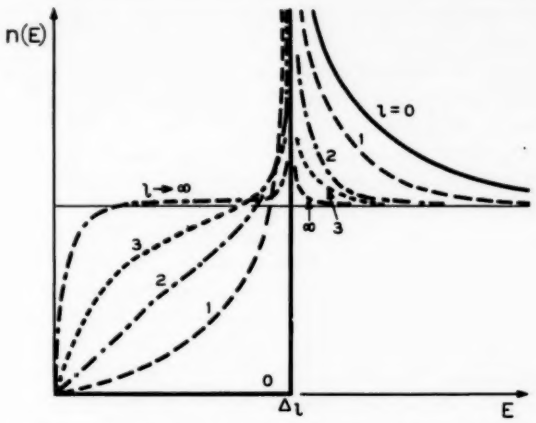

FIG. 1. Density of states at $T=0$ as a function of energy for various values of l.

by calculating the correlation:

$$\langle nn(\vec{\tau})\rangle = \langle 0|n(\vec{r})n(\vec{r}+\vec{\tau})|0\rangle_{\text{av}}. \quad (9)$$

In our generalized state we find a similar "hump," containg $1.57\epsilon_0/\epsilon_F$ electrons, but we also find that the "hump" is rotating. That is, we can calculate

$$\langle n\vec{j}(\vec{\tau})\rangle = \langle 0|n(\vec{r})\vec{j}(\vec{r}+\vec{\tau})|0\rangle_{\text{av}}. \quad (10)$$

This is nonvanishing and has the value

$$[\langle n\vec{j}(\vec{\tau})\rangle\times\vec{\tau}]\cdot\hat{n} = \langle nn\rangle\, 2l\hbar; \quad (11)$$

that is to say, each particle in the "hump" has angular momentum $2l\hbar$ about the arbitrary spatial direction $\hat{n}$.

If we now calculate the currents in a bounded sample by means of

$$\langle\vec{j}(r)\rangle = \frac{\mathrm{Tr}\,\vec{j}(r)\exp[-\beta(\mathcal{H}-\mu N-\delta H_b)]}{\mathrm{Tr}\exp[-\beta(\mathcal{H}-\mu N-\delta H_b)]}, \quad (12)$$

where δH_b is a perturbation introduced at the boundary to confine the liquid, it is easily seen that (11) gives a surface current at the boundary just such as to imply a total angular momentum of the liquid $\sim n_c l\hbar$, where n_c is the effective number of paired particles. The only analogy to this we can draw is to the alignment of electron spins in a ferromagnet; we seem here to have

ferromagnetically aligned orbital angular momentum.

VII. Less spectacular are the superfluidity (or superconductivity) properties. As pointed out by Glassgold and Sessler,[7] these states are superfluid (or in a metal superconducting) with somewhat anisotropic superfluidity properties. As in V, the only striking differences will be near $T=0$, where such properties as the normal density or (for the superconductor case) the difference from the London penetration depth will vary as powers of $T^{1/l}$ instead of exponentially. We find the penetration depth difference, for example, to go as $T^{4/l}$.

VIII. Unlike the case of $l=0$,[8] the states for all higher l's are sensitive to impurity scattering effects. This means that the mean free path must be long compared to ξ_0, which may explain the failure to observe the effect as yet.

We should like especially to acknowledge early discussions on this work with K. A. Brueckner and B. T. Matthias. We are also indebted for discussions and prepublication copies of their work to A. E. Glassgold, V. J. Emery, and A. M. Sessler.

[1]J. C. Fisher (private communication); D. J. Thouless (to be published).
[2]K. A. Brueckner, T. Soda, P. W. Anderson, and P. Morel, Phys. Rev. 118, 1442 (1960).
[3]V. J. Emery and A. M. Sessler, Phys. Rev. 119, 43 (1960).
[4]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).
[5]P. W. Anderson, Phys. Rev. 112, 1900 (1958).
[6]N. N. Bogolyubov, V. V. Tolmachev, and D. V. Shirkov, New Method in the Theory of Superconductivity, (Academy of Sciences of U.S.S.R., Moscow, 1958), Chap. 6.3.
[7]A. E. Glassgold and A. M. Sessler (private communication).

THERMAL CONDUCTIVITY OF ISOTOPIC MIXTURES OF SOLID HELIUM

F. W. Sheard and J. M. Ziman
Cavendish Laboratory, University of Cambridge, Cambridge, England
(Received July 21, 1960)

The thermal conductivity of solid He^4 containing small percentages of He^3 has been measured by Walker and Fairbank,[1] who argued that the observed isotope resistivity was so much larger than the value deduced from the theory of Klemens[2] that the He^3 atoms must in some way be concentrated along lines, rather than being uniformly distributed through the solid. But their results are actually in good agreement with values calculated from the variational treatment.[3] In fact, the isotope scattering is so small that we are in the region where phonon-phonon N-processes are dominant, and can use the simple limiting formula[4] which gives a resistivity exactly 55 times larger than the Klemens formula. So, in Fig. 5 of Walker and Fairbank's paper, we simply raise the theoretical curve labelled "Eq. (3)" by a factor 55—which takes it almost exactly into the middle of the observed data. Moreover, we can understand why the resistivity per atomic fraction of He^3 tends to be rather smaller at higher concentration—this is exactly what is predicted by the variational theory as we move away from the region where isotope scattering is small in comparison with N-process. It is true that we cannot explain why the isotope resistance appears to be nearly independent of the temperature in this range, but, as pointed out by Walker and Fairbank, one cannot put very much weight upon results obtained after simply subtracting umklapp resistance as if Matthiessen's law were exactly true. There certainly does not seem any need to invoke lines of He^3 atoms to explain these observations.

[1]E. J. Walker and H. A. Fairbank, Phys. Rev. 118, 913 (1960).
[2]P. G. Klemens, Proc. Roy. Soc. (London) A208, 108 (1951).
[3]R. Berman, P. T. Nettley, A. N. Spencer, R. W. H. Stevenson, F. W. Sheard, and J. M. Ziman, Proc. Roy. Soc. (London) A253, 408 (1959).
[4]J. M. Ziman, Can. J. Phys. 34, 1256 (1956); Electrons and Phonons (Oxford University Press, New York, 1960).

THERMAL CONDUCTIVITY OF SOLID He^3† *

E. J. Walker and Henry A. Fairbank
Yale University, New Haven, Connecticut
(Received July 25, 1960)

Solid He^3 has been found to exist in two solid phases.[1] The lower density, lower pressure phase, α, is body-centered cubic[2]; the higher density, higher pressure phase, β, has an hexagonal close-packed structure[2] as does solid He^4. We wish to report experimental measurements of the thermal conductivity of both phases of solid He^3 above 1.1°K for several different densities. The thermal conductivity of solid He^3 in the β phase is similar to that of solid He^4, [3,4] and in reasonable agreement with that expected on Peierls' theory for umklapp scattering. Of particular interest are the results in the α phase of solid He^3 where the thermal conductivity exhibits an anomalous behavior below about 1.5°K.

Measurements were made on samples of solid helium kept at constant density using the apparatus and method of measurement previously described.[4] The experimental results for solid He^3 are shown in Fig. 1, where the thermal conductivity K is plotted on a logarithmic scale against $1/T$ for six different densities. The data of densities A and B are fitted to straight lines and C-F to smoothed curves. Since each measurement was taken at constant density the results are most easily discussed by reference to the density diagram of Fig. 2.[1] The equilibrium density of the α and β solids along the phase boundary and that of solid and liquid along the melting curve are shown. The measurements in Fig. 1 were taken along the portions of the dotted lines A-F in Fig. 2 lying in the solid region. For curve A the solid He^3 is in the β phase throughout. It is clear that the discontinuity and change in slope in curve B, Fig. 1, occurs at the α - β phase transition, the β phase exhibiting the higher conductivity. In curves C through F, which appear to lie entirely

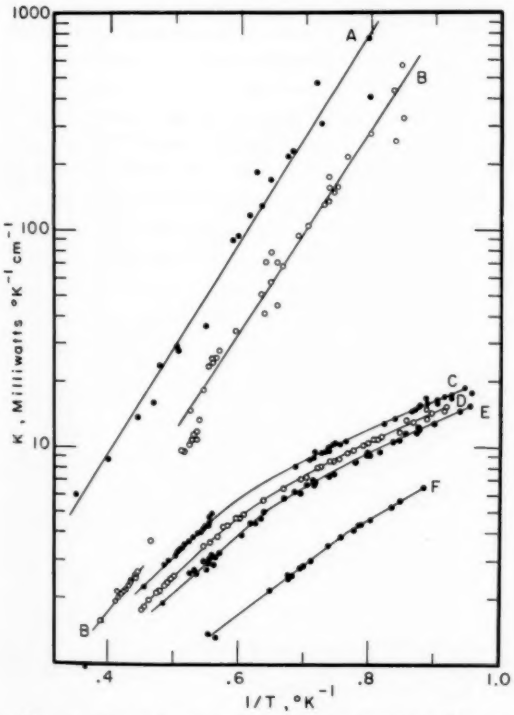

FIG. 1. The thermal conductivity of solid He^3 as a function of the reciprocal temperature for the densities indicated in Fig. 2.

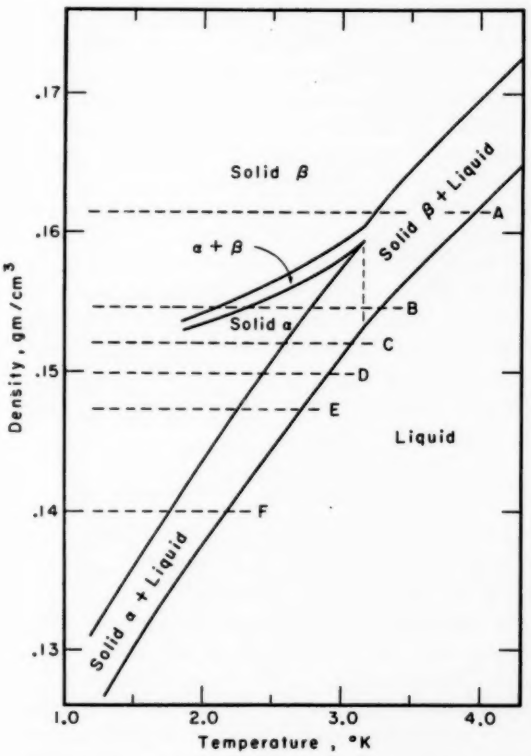

FIG. 2. The phase equilibrium density vs temperature diagram of solid He^3.[1] The dashed lines indicate the densities at which the corresponding curves A-F in Fig. 1 were taken.

within the α phase, a change in slope occurs at temperatures between 1.3°K and 1.6°K. There is a striking difference at lowest temperatures (nearly a factor of 50 at 1°K) between the conductivities in the α and β phases.

The results in the β phase are similar to those obtained in pure He^4, [3,4] and are consistent with the assumption that the thermal resistance of the crystal arises principally from phonon-phonon umklapp interactions. For this case Peierls[5] has shown that

$$K = AT^n \exp(\theta/bT), \qquad (1)$$

where A is a constant, n and b are constants of order unity, and θ is the Debye characteristic temperature. The data of curve A, Fig. 1, fits the equation $K = 0.10 \exp(11.1/T)$ milliwatts $(°K)^{-1}$ cm^{-1} in agreement with Eq. (1) with n very small.

To account for the behavior of curves C-F, several mechanisms might be considered. One possibility is the existence of a new low-temperature phase in solid He^3. For several reasons this seems unlikely. We would expect a change in θ and in K in going from the α phase to the new phase as was observed in the α - β transition. Such a shift in K was not observed.

Another possibility is that θ/b is a linear function of T for each density in the temperature range of the measurements. The data of curves C through F are consistent with this model if the curve of θ/b vs T changes slope at the temperature at which the change in slope occurs in the curves of Fig. 1. This would require a suitable dependence of the elastic constants on temperature.

Another possible explanation is that two groups of phonons with different excitation energies are involved in the umklapp processes of the α phase, giving rise to a thermal conductivity of the form

$$\frac{1}{K} = \frac{1}{A_1 \exp(T_1/T)} + \frac{1}{A_2 \exp(T_2/T)}.$$

Curves C-F can be fitted reasonably well by this relation. For curve C, $A_1 = 5.36 \times 10^{-2}$ mw/cm°K, $A_2 = 2.94$ mw/cm°K, $T_1 = 9.0$°K, $T_2 = 2.0$°K. In a body-centered cubic lattice in which only central forces exist, shear motions of the (110) planes in the $\langle 1\bar{1}0 \rangle$ directions do not change nearest neighbor distances and hence a bcc lattice is unstable in the absence of second-nearest neighbor interactions. The large zero-point motion in solid He^3 would be expected to increase the second-nearest neighbor interactions, which could account for the stability of the bcc lattice. He^3 is, in fact, the only dielectric element known to exist as a stable solid in the bcc lattice. The frequencies of transverse phonons travelling in or near the (110) direction, polarized in the $\langle 1\bar{1}0 \rangle$ directions, should be substantially lower than those in other directions. These low-lying branches of the phonon spectrum could provide the lower excitation energy implied by the decreased slope in the α-phase curves of Fig. 1 at low temperatures.

With reference to this suggested mechanism, it is interesting to note that Sydoriak, Mills, and Grilly[6] have recently presented evidence for a density maximum in solid He^3 slightly above 1°K along the melting curve. Thus it appears probable that the expansion coefficient changes sign in the vicinity of the changes in slope of curves C-F in Fig. 1. The slopes of these curves are seen to increase with increasing density at higher temperatures and to decrease with increasing density at lower temperatures, which would suggest a corresponding density dependence for the dominant umklapp frequencies. A similar trend of the frequencies of the most important thermal vibrations would imply a change in sign of the expansion coefficient. The decrease of the frequency of the low-lying branches of the phonon spectrum with pressure could conceivably occur since the zero-point motion and hence the strength of second-nearest neighbor interactions should decrease with increasing density.

We are deeply indebted to Professor Lars Onsager for several stimulating discussions and suggestions on the interpretation of these data.

---

†Assisted by the Office of Naval Research.

*Part of this work has been submitted to Yale University by E. J. Walker in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1]E. R. Grilly and R. L. Mills, Ann. Phys. 8, 1 (1959).

[2]A. F. Schuch, E. R. Grilly, and R. L. Mills, Phys. Rev. 110, 775 (1958).

[3]F. J. Webb, K. R. Wilkinson, and J. Wilks, Proc. Roy. Soc. (London) A214, 546 (1952).

[4]E. J. Walker and H. A. Fairbank, Phys. Rev. 118, 913 (1960).

[5]R. E. Peierls, Quantum Theory of Solids (Oxford University Press, New York, 1956).

[6]S. G. Sydoriak, R. L. Mills, and E. R. Grilly, Phys. Rev. Letters 4, 495 (1960).

CONTAINMENT OF POSITRONS IN A MIRROR MACHINE*

Gordon Gibson,† Willard C. Jordan,‡ and Eugene J. Lauer
Lawrence Radiation Laboratory, University of California, Livermore, California
(Received July 11, 1960)

A significant fraction of the positrons emitted isotropically from a uniform volume source has been contained in a static magnetic mirror machine for a mean containment time greater than 10 sec; this corresponds to more than 10^{10} cyclotron periods. Aside from its fundamental interest, this is of practical importance in various research fields such as controlled fusion and studies of the Van Allen radiation belt. It may also be of value in the study of the particle which is kept under observation.

The apparatus, which is shown in Fig. 1, consists of a magnetic mirror machine (coaxial 60-in. mean-diameter coils with 64 in. between coil centers) located adjacent to the Livermore cyclotron used to produce Ne^{19}. The radioactive gas is introduced into a 20-in. diameter vacuum tank where it provides a source of positrons and from which it may be rapidly removed by opening a 20-in. diameter valve.[1] This valve isolates the vacuum system from a 32-in. oil diffusion pump (base pressure $\simeq 10^{-7}$ mm Hg). Noble gases which serve as known scattering gases can be introduced in such a manner that their partial pressure is independent of the position of the fast-opening valve. Intermittent evaporation of titanium maintains a relatively small base pressure ($\simeq 10^{-8}$ mm Hg) of an unknown gas when the fast valve is closed, but does not pump on the Ne^{19} or other noble gases. The rate of escape of positrons through one mirror is monitored by a scintillation counter located just outside a thin foil window in the end wall of the vacuum chamber. For the data presented, the 1.5-in. diameter sensitive area of the counter was on axis. Electronic circuitry provides for the time analysis of the pulses which are within a selected range in amplitude, corresponding to a known energy range. The calibration (pulse height vs positron energy) and resolution of the counter are obtained using monoenergetic electrons from a beta-ray spectrometer.

Positrons which are created with their velocity vector in either escape cone leave promptly without reflecting. Those which are not in either escape cone are initially trapped and reflect back and forth until their precessional surfaces intercept the wall because of asymmetries, or until they escape due to nonadiabatic effects or gas scattering. If multiple small-angle scattering is the dominant effect, and the Larmor radius is small relative to the vacuum tank radius, then most of the trapped component escapes via the weaker mirror when the two mirrors are not of

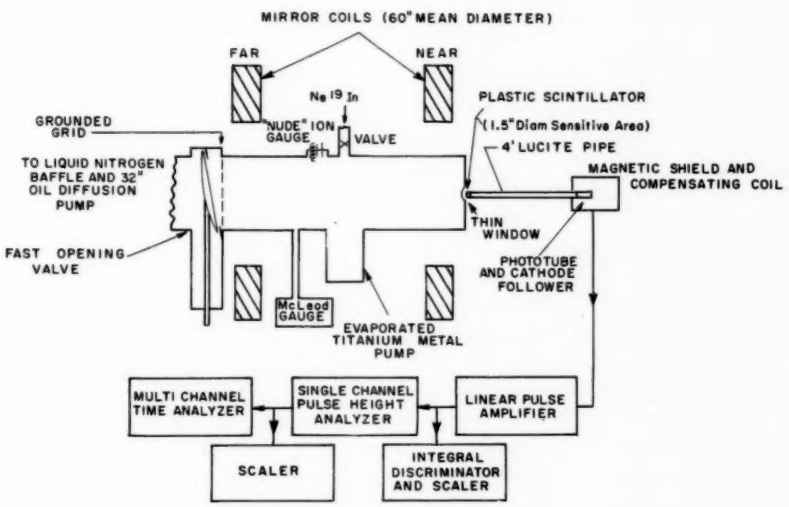

FIG. 1. Schematic diagram of apparatus.

equal strength (i.e., bias $\neq 0$). The bias is defined as $2(I_f - I_n)/(I_f + I_n)$, where I_f and I_n are the currents through the mirror coils, which are far from and near to the counter, respectively. In collecting a set of data at various bias values $(I_f + I_n)$ was held constant.

To start a "fast-empty" run, a valve in the Ne^{19} source line is closed and the trapped positron density ($< 10^4$ cm^{-3}) is allowed to come to equilibrium with the Ne^{19} density. At time zero the large valve (opening time = 0.04 sec) is opened and the Ne^{19} diffuses into the pump and is removed with a 0.3-sec e-fold time (measured by observing the counting rate vs time with the field off). Removal of the Ne^{19} causes a sudden decrease of counting rate due to removal of the prompt component, followed by a slower decrease due to the escape of the trapped positrons. Figure 2(a) shows a typical set of data obtained with stable neon added as the scattering gas. The logarithm of the counting rate is plotted versus time for three different pressures of neon. The curves are normalized to have a counting rate of unity at zero time. Following the initial drop there is a region in time which can be fitted by a straight line within the uncertainty set by counting statistics. The time constant, τ, in this region of the curve is taken as a measure of the containment time and the intercept of this straight line at zero time is the fraction of the counting rate at equilibrium which is due to these trapped particles. The observed times are listed in Table I.

The theoretical values in Table I (column 3) are based on the following formula which is derived from gas scattering theory[2]:

$$\tau = \frac{\beta^3 \langle \theta^2 \rangle_{av}}{8\pi N_0 (Z^2 + Z) r_0^2 c (1 - \beta^2) \ln(\delta/\Delta)},$$

where $\langle \theta^2 \rangle_{av} = 2[\pi/2 - \sin^{-1}(1/\sqrt{R})]^2$, R = maximum field/minimum field (for bias $\neq 0$, the smaller of the two R values is to be used), N_0 = scattering gas density (atoms/cm^3), $\delta = 0.1$, $\Delta = [2Z^{4/3}\alpha^2 \times (1 - \beta^2)^{1/2}]/\beta^2$, and the other symbols have their usual meaning. Column 4 of Table I gives the theoretical times after correction for the effects of energy degradation of the beta spectrum and energy resolution of the detection system. The distribution in energy of the trapped particles, $N(E)$, and their leakage rate, $N(E)/\tau(E)$, are computed as functions of time. The counting rate is obtained by integration of the leakage rate over the resolution of the detector. It is assumed that the energy loss is due to interactions with the scattering gas atoms, and that the particles behave adiabatically until they escape and hit the counter because of multiple Coulomb scattering. Values of τ obtained with neon scattering gas pressure as low as 10^{-7} mm Hg scale correctly

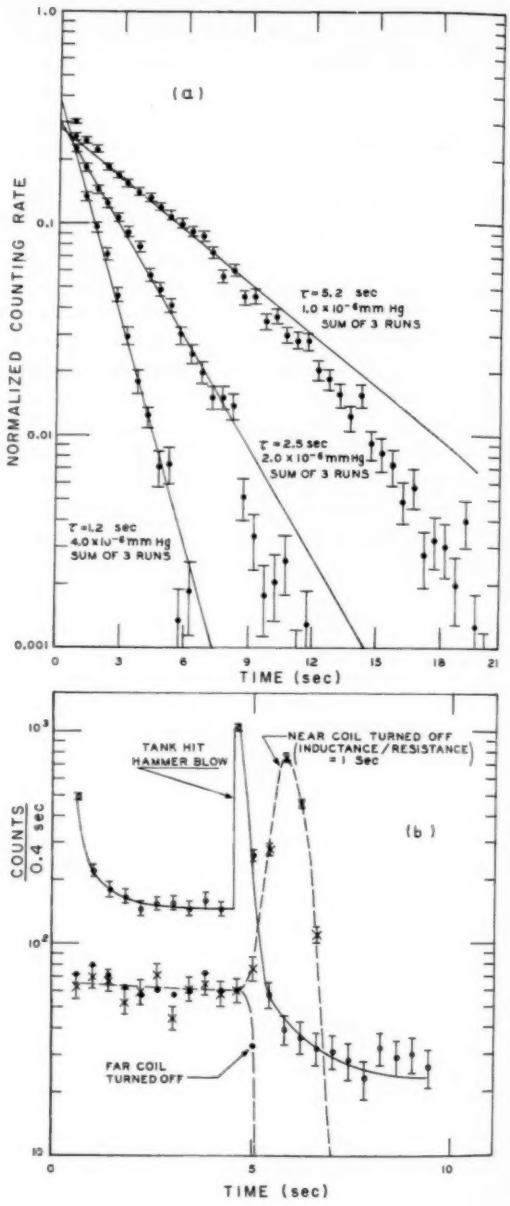

FIG. 2. Counting rate vs time after opening fast valve, for central field of 1300 gauss, zero-bias mirror ratio of 1.8. (a) 1.00 ±0.23 Mev, neon scattering gas, base vacuum 5×10^{-8} mm Hg, bias 13%. (b) 0.50 ±0.15 Mev, bias approx. 1%.

Table I. Observed mean containment times (τ) compared with the predictions of gas scattering theory, for central field of 1300 gauss, bias of +13%, and R equal to 1.68.

Energy setting and resolution (Mev)	Pressure of neon (10^{-6} mm Hg)	τ (sec) Theoretical (single energy)	τ (sec) Theoretical (corrected)	τ (sec) Observed
0.50 ± 0.15	1.0	2.7	5.9	5.4
	2.0	1.4	3.0	2.9
	4.0	0.68	1.5	1.3
1.00 ± 0.23	1.0	7.0	5.0	5.2
	2.0	3.5	2.5	2.5
	4.0	1.7	1.3	1.2

with pressure (within 30%) but require additional corrections for radiation effects.[3] The more rapid dropoff of counting rate observed at long times is expected from the theoretical treatment of the energy loss.

The gas pressure is read on an ion gauge which is calibrated by an expansion technique using a McLeod gauge. Repeated calibrations indicate an uncertainty of ± 5%. Repeated measurements of the containment time yield maximum deviations of ± 10%. The observations in Table I scale correctly with pressure and agree satisfactorily in magnitude with the corrected theoretical values.

Trapped particles may be "dumped" by mechanically shocking the vacuum tank [see Fig. 2(b)]. It is thought that dust falling through the contain-

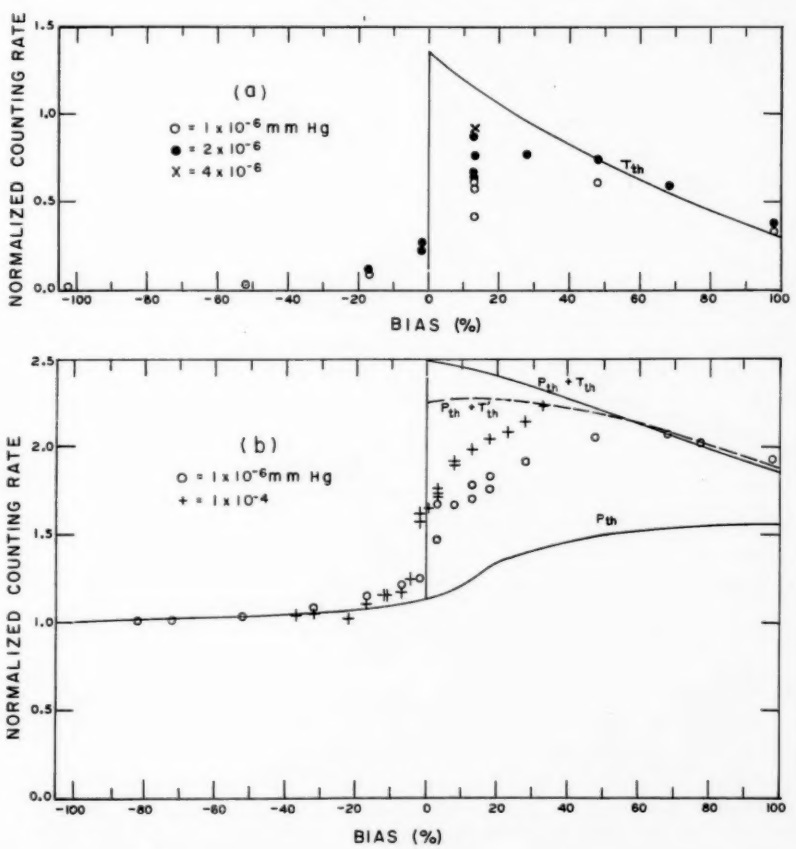

FIG. 3. Theoretical and observed counting rates as a function of bias, for central field of 1300 gauss, zero-bias mirror ratio of 1.8, 1.00 ± 0.23 Mev, neon scattering gas. (a) Counting rate due to particles having long mean containment time (corrected for energy loss) compared with theory. (b) Total equilibrium counting rate compared with theory.

ment region causes a large fraction of the trapped positrons to scatter and escape through a mirror. The remaining fraction escapes with the characteristic scattering-out time. Also shown in Fig. 2(b) are the effects of turning off the near or far coil.

The relative counting rate due to positrons which are initially trapped (T_{th}) and those which escape promptly (P_{th}) (normalized to unity at -100% bias) has been calculated numerically as a function of bias for the steady state, by assuming that all the trapped positrons behave adiabatically and escape from the weaker mirror. In Fig. 3(a) the zero time intercepts of the "fast-empty" curves have been corrected for energy degradation effects and are compared with T_{th}. Normalization of these data has been achieved by measuring the total equilibrium counting rate prior to opening the large valve. In an equilibrium experiment, the source gas is held in the closed chamber and the counting rate is measured as a function of bias. In Fig. 3(b) the experimental results are compared with $(P_{th} + T'_{th})$, which is the theoretical total counting rate corrected for energy degradation effects. The different measurements are normalized to the same source-gas density by following each measurement with a measurement of the number of counts received in an accurately timed interval with the magnetic field off. Comparing Figs. 3(a) and 3(b) it is seen that, with the experimental uncertainty, all of the trapped particles escape with the mean containment time predicted by scattering theory. A fraction of the trapped particles is detected on negative bias. This is due to a sudden change in pitch angle, greater than the difference between the two escape angles, during the time it takes for multiple small-angle scattering to accumulate to a large angle.[4] Deviations from the theoretical curve may also be due to small asymmetries since it has been observed that tilting a coil slightly changes the observed bias curve drastically.

The present results agree with the "fast filling" experiment.[5] They also appear to agree with, and extend, the conclusions of an experiment in which ionization currents created by 18-kev electrons in a mirror machine were measured.[6]

We are pleased to acknowledge the help of J.D. Kinney, J. L. Murphy, and the operators of the Livermore 90-in. cyclotron, the discussions with T. G. Northrop, and the support of C. M. Van Atta.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Visiting from Westinghouse Atomic Power Department, Pittsburgh, Pennsylvania.

‡Visiting from Bendix Corporation, Research Laboratories Division, Detroit, Michigan.

[1]Designed by T. H. Batzer and J. R. Ullman of this laboratory.

[2]G. Gibson and E. J. Lauer, University of California Lawrence Radiation Laboratory Report UCRL-5696 (unpublished).

[3]G. Gibson and E. J. Lauer, Phys. Rev. 117, 1188 (1960).

[4]This may provide a means for observing single scattering events that would normally be masked by the multiple small-angle scattering.

[5]G. Gibson and E. J. Lauer, Bull. Am. Phys. Soc. 3, 412 (1958).

[6]S. N. Rodionov, Atomnaya Energiya 6, 623 (1959).

GROUND-STATE SPLITTING FOR d^5 ^6S IONS IN A CUBIC FIELD

M. J. D. Powell, J. R. Gabriel, and D. F. Johnston
Atomic Energy Research Establishment, Harwell, England
(Received July 1, 1960)

The calculations of Watanabe[1] have been used to interpret paramagnetic resonance experiments on d^5 ^6S ions by Walsh[2] and others.[3-5] Watanabe omitted the spin-doublet states of the configuration d^5, and treated the crystal field, spin-orbit, and spin-spin interactions as perturbations of the free-ion Hamiltonian.

By using a Ferranti Mercury computer and making full use of group theory, we have been able to perform a more refined calculation. Our results differ very considerably from those of the Watanabe theory.

Our calculation includes the spin-doublet states of d^5, treats the crystal field as part of the unperturbed Hamiltonian, and the spin-orbit W_{SO} and spin-spin W_{SS} interactions as perturbations. The perturbation calculation was taken to sixth order. We calculate the following four cases.

(1) Splitting by $(W_{SO}+W_{SS})$ taking account of the spin doublets.

(2) As in (1) but omitting the doublet states.

(3) As in (1) but omitting W_{SS}.

(4) As in (1) but omitting doublets and W_{SS}.

As an example, taking $B=0.9\times10^3$ cm^{-1}, $C=3.3\times10^3$ cm^{-1} for the Racah[6] parameters, $\lambda=0.4\times10^3$ cm^{-1} for the spin-orbit parameter, and $M_0=0.2042$ cm^{-1} and $M_2=0.0159$ cm^{-1} for the spin-spin parameters (calculated from Watson's[7] wave functions for Mn^{++}), and a range of values for the cubic field parameter Dq, we compute the values of the ground-state splitting $3a=E(\Gamma_8)-E(\Gamma_7)$ given in Table I.

There are several points of interest in these results:

(a) Omitting the spin-doublets of d^5, as Watanabe did, reduces the calculated splitting by almost two orders of magnitude.

(b) From a consideration of the variation of the splitting with Dq for practical values of Dq, it is clear that the effect of the cubic field cannot be approximated by second order terms with fourth order corrections, as has been suggested.[1]

(c) The splitting is not an even function of Dq. Watanabe's argument that only even terms occur fails because on going from a description in terms of electrons to one in terms of holes, the signs of both λ and Dq must be reversed. Therefore functions which are odd in both parameters cannot be excluded. However, we agree with Watanabe that the original suggestion of Van Vleck and Penney[8] must be incorrect because it is odd in Dq and even in λ.

(d) For practical Dq values, the contribution from spin-spin interaction is negligible compared to that from spin-orbit interaction. However, in the approximation of neglecting doublets we agree

Table I. Calculated ground-state splitting in Mn^{++}, in units of 10^{-4} cm^{-1}, for given parameters.

Dq cm^{-1}	With spin-spin		Without spin-spin	
	With doublets	Without doublets	With doublets	Without doublets
+1200	89.7	0.971	80.2	-2.08
+1000	46.8	0.784	42.1	-1.08
+ 800	23.1	0.513	21.1	-0.547
+ 600	9.81	0.264	9.45	-0.260
+ 400	2.65	0.0824	3.06	-0.107
+ 200	-0.335	-0.00903	0.178	-0.0301
0	0	0	0	0
- 200	3.56	0.115	2.40	-0.00825
- 400	11.0	0.338	7.86	-0.0622
- 600	23.8	0.668	17.7	-0.188
- 800	45.1	1.09	34.4	-0.443
-1000	81.4	1.59	63.6	-0.937
-1200	146	2.07	117	-1.89

with Watanabe that spin-spin is more important than spin-orbit, although we do not agree quantitatively with his calculations because he has used the large value of $M_0 = 0.642$ and also appears to have used an incorrect perturbation formula.

The main uncertainty in a comparison with experiment is due to the uncertainty in the values of the parameters B, C, λ, and Dq, the calculated splitting being particularly sensitive to C, λ, and Dq. Because of this it is very desirable that optical and electron paramagnetic-resonance experiments should be done on the same material. This uncertainty affects the value of Dq which we calculate from a given a. However, Low has reported the following ground-state splittings: $3a = 56\times10^{-4}$ cm^{-1} for Mn^{++} in MgO, $3a = 24\times10^{-4}$ cm^{-1} for Mn^{++} in ZnS, and $3a = 615\times10^{-4}$ cm^{-1} for Fe^{+++} in MgO.

Assuming the B, C, and λ values used to calculate Table I, we find $Dq = +1050$ cm^{-1} for Mn^{++} in MgO and $Dq = -600$ cm^{-1} for Mn^{++} in ZnS; and for Fe^{+++} (taking free ion values $B = 1.1\times10^3$ cm^{-1}, $C = 4.0\times10^3$ cm^{-1}, together with Kotani's[9] value $\lambda = 0.44\times10^3$ cm^{-1}, and $M_0 = 0.2902$ cm^{-1}, $M_2 = 0.0228$ cm^{-1} from Watson's wave functions) we calculate $Dq = 2150$ cm^{-1}. The predicted Dq value for Mn^{++} in MgO agrees with Pratt and Coelho's[10] value of $Dq = 979$ cm^{-1} in MnO which has the same structure and almost the same lattice constant. (Pratt and Coelho give values of B, C, and Dq which lead to $3a = 60\times10^{-4}$ cm^{-1}.) The values for Mn^{++} in ZnS and Fe^{+++} in MgO agree roughly with the conclusions of Orgel,[11] and the ratio of Dq values for Mn^{++} and Fe^{+++} in MgO is the same as that found by Holmes and McClure[12] in hydrated salts. When optical and electron paramagnetic resonance data on the same material become available, it will be possible to apply the more stringent test of comparing calculated and experimental values of the splitting.

In comparing with Walsh's experimental values of $(\partial\ln a/\partial\ln V)_T$, large uncertainties arise because we have no proper theory to predict $\partial\ln Dq/\partial\ln V$, $\partial\ln\lambda/\partial\ln V$, etc. The simple ionic model gives $\partial\ln Dq/\partial\ln V = -5/3$, but since the model is probably not reliable and we have no values of $\partial\ln\lambda/\partial\ln V$, etc., we can only calculate values of $\partial\ln Dq/\partial\ln V$ from the experimental results and our theory, assuming that λ, B and C remain constant under dilatation.

With this assumption our deductions from Walsh's results are $\partial\ln Dq/\partial\ln V = -2.1$ for Mn^{++} in MgO, $\partial\ln Dq/\partial\ln V = -1.6$ for Mn^{++} in ZnS, and $\partial\ln Dq/\partial\ln V = -0.9$ for Fe^{+++} in MgO.

To conclude, the values of Dq which we deduce are in as good agreement with experiment as can be expected in view of the uncertainties in the model, but using our theory and the experimental results of $\partial\ln a/\partial\ln V$ we deduce various values of $\partial\ln Dq/\partial\ln V$ which are quite different from the values of $-5/3$ predicted by an ionic model.

We wish to thank Dr. W. Marshall for many helpful discussions, and Dr. J. M. Soper for an independent check of Marvin's[13] results for the matrix elements of the spin-spin interaction within the configuration d^2.

[1]H. Watanabe, Prog. Theoret. Phys. (Kyoto) 18, 405 (1957).

[2]W. M. Walsh, Phys. Rev. Letters 4, 507 (1960).

[3]W. Low, Paramagnetic Resonance in Solids, (Academic Press, New York, 1960), p. 118.

[4]H. Watanabe, Phys. Rev. Letters 4, 410 (1960).

[5]J. Baker, B. Bleaney, and W. Hayes, Proc. Roy. Soc. (London) A247, 141 (1958).

[6]G. Racah, Phys. Rev. 62, 438 (1942).

[7]R. E. Watson, Massachusetts Institute of Technology Technical Report, No. 12, 1959 (unpublished).

[8]J. H. Van Vleck and W. G. Penney, Phil. Mag. 17, 961 (1934).

[9]M. Kotani, J. Phys. Soc. (Japan) 4, 293 (1949).

[10]G. W. Pratt and R. Coelho, Phys. Rev. 116, 281 (1959).

[11]L. E. Orgel, J. Chem. Phys. 23, 1004 (1955).

[12]O. G. Holmes and D. S. McClure, J. Chem. Phys. 26, 1686 (1957).

[13]H. H. Marvin, Phys. Rev. 71, 102 (1947).

ENERGY GAP IN SUPERCONDUCTORS MEASURED BY ELECTRON TUNNELING

Ivar Giaever
General Electric Research Laboratory, Schenectady, New York
(Received July 5, 1960)

If a potential difference is applied to two metals separated by a thin insulating film, a current will flow because of the ability of electrons to penetrate a potential barrier. The fact that for low fields the tunneling current is proportional to the applied voltage[1] suggested that low-voltage tunneling experiments could reveal something of the electronic structure of superconductors.

Aluminum/aluminum oxide/lead sandwiches were prepared by vapor-depositing aluminum on glass slides in vacuum, oxidizing the aluminum in air for a few minutes at room temperature, and then vapor-depositing lead over the aluminum oxide. The oxide layer separating aluminum and lead is thought to be about 15-20A thick.

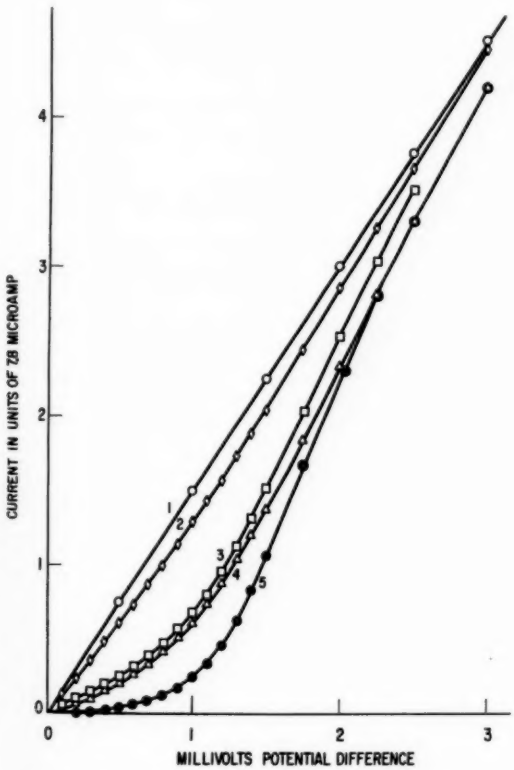

FIG. 1. Tunnel current between Al and Pb through Al_2O_3 film as a function of voltage. (1) $T = 4.2°K$ and $1.6°K$, $H = 2.7$ koe (Pb normal). (2) $T = 4.2°K$, $H = 0.8$ koe. (3) $T = 1.6°K$, $H = 0.8$ koe. (4) $T = 4.2°K$, $H = 0$ (Pb superconducting). (5) $T = 1.6°K$, $H = 0$ (Pb superconducting).

At liquid helium temperature, in the presence of a magnetic field applied parallel to the film and sufficiently strong to keep the lead in the normal state, the tunnel current is linear in the voltage. However, when the magnetic field is removed, and lead becomes superconducting, the tunnel current is very much reduced at low voltages as shown in Fig. 1. There is no influence of polarity, identical results being obtained with both directions of current flow.

The slope dI/dV of the curve in Fig. 1 where $H = 0$, $T = 1.6°K$, divided by dI/dV for normal lead, is plotted in Fig. 2. On the naive picture that tunneling is proportional to density of states,[2] this curve expresses the density of states in superconducting lead relative to the density of states when lead is in its normal state, as a function of energy measured from the Fermi energy. It seems clear that the density of states at the Fermi level is drastically changed when a metal becomes a superconductor, the change being symmetric with respect to the Fermi level. The curve resembles the Bardeen-Cooper-Schrieffer[3] density of states for quasi-particle excitations. There is a broadening of the peak that decreases with decreasing temperature. An approximate measure of half the energy gap

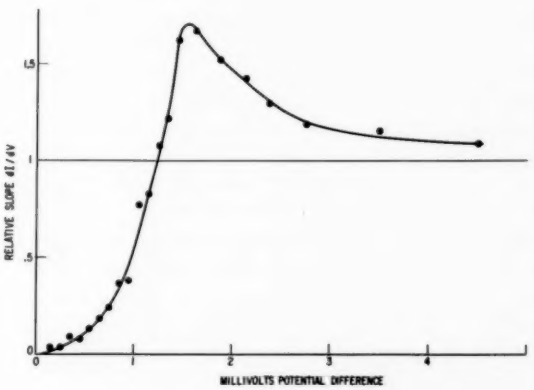

FIG. 2. From Fig. 1, slope dI/dV of curve 5 relative to slope of curve 1.

is given by the point at which the relative slope $dI/dV = 1$. On this basis the gap width for lead is $(4.2 \pm 0.1)kT_c$.

The experiment has been repeated with tin and indium giving entirely similar results; the gap in each case is approximately $4kT_c$. These results are of a preliminary nature, and experiments at lower temperatures will make them more precise.

I wish to thank C. P. Bean and J. C. Fisher for their interest and encouragement, and P. E. Lawrence for his help in performing the experiments.

[1]J. C. Fisher and I. Giaever (to be published).

[2]W. A. Harrison (private communication) has pointed out that the tunnel current is not proportional to the density of states except in the limiting case of a low density of states.

[3]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

CRITICAL FIELD FOR SUPERCONDUCTIVITY IN NIOBIUM-TIN

R. M. Bozorth, A. J. Williams, and D. D. Davis
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received July 20, 1960)

It is well known[1] that Nb_3Sn is a superconductor with a high critical temperature, 18°K. The measurements here reported show that it has also an exceptionally high critical field, about 70 000 oersteds at 4.2°K, necessary for the suppression of all superconductivity.

The material was prepared by melting together niobium and tin in the argon arc, and the button so obtained was formed by grinding into a rod about 2 cm long and 4 mm in diameter, with rounded ends. The magnetic moment per gram, σ_g, was measured by pulling the specimen from one search coil to another in a constant field, the two search coils being connected in series opposition to a ballistic galvanometer. Calibration was with nickel of high purity.

Measurements were made in increasing fields, after cooling in zero field to liquid helium temperature. Results are shown in Fig. 1. The initial points (circles) follow accurately the line for $B = 0$ ($H = -4\pi\sigma_g d$, where d is the density, 8.9), and then begin to deviate at about 4000 to 5000 oersteds. The variations in the readings in fields from 5000 to 20 000 oersteds reflect the well-known irregular changes in magnetization resulting from changes in domain structure in the intermediate state, as observed by Schawlow et al.[2] and others. The general shape of the magnetization curve is that observed in a hard superconductor. Polishing, or annealing the specimen at 1100°C for several hours, made no essential change in the character of the curve.

When the field was decreased from its maximum value (points marked with squares) some of the flux was frozen in, and irregularities were again observed.

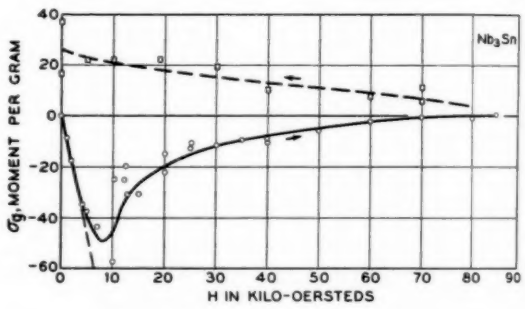

FIG. 1. Magnetization of Nb_3Sn as dependent on field strength, showing superconduction in entire specimen to about 5000 oersteds and superconduction in some parts of specimen to about 70 000 oersteds.

The authors are indebted to E. Corenzwit for preparation of the material, to W. E. Henry of the Naval Research Laboratory for details of the method of measurement, and to H. W. Dail for assistance with the experiment. The field was produced in a Bitter coil excited with a motor generator with a nominal power rating of one megawatt.

[1]B. T. Matthias and T. H. Geballe, Phys. Rev. 95, 1435 (1954).

[2]A. L. Schawlow, G. E. Devlin, and J. K. Hulm, Phys. Rev. 116, 626 (1959).

CORRELATIONS BETWEEN SUPERCONDUCTIVITY AND NUCLEAR MAGNETIC RESONANCE PROPERTIES

W. E. Blumberg, J. Eisinger, V. Jaccarino, and B. T. Matthias
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received July 27, 1960)

Using nuclear magnetic resonance (NMR) techniques, we have measured the Knight shifts,[1] linewidths, and quadrupole interactions in a series of vanadium intermetallic compounds V_3X (where X = As, Au, Co, Ga, Ge, Ir, Pt, Sb, Si),[2] all having the β-wolfram structure.[3] These experiments reveal certain striking correlations between the NMR properties and the superconducting transition temperatures of these compounds. Measurements were made with a Varian NMR spectrometer in a temperature range between 4°K and 400°K using 400-mesh particles.

Vanadium Knight shifts k_V, which were measured relative to KVO_3 solution, are shown as functions of temperature in the upper part of Fig. 1. While Knight shifts normally are expected to be temperature independent, several of these V_3X compounds show a marked temperature dependence of k_V. This temperature dependence is several orders of magnitude greater than and in the opposite direction from that expected from lattice contraction. The compounds with the highest superconducting transition temperature T_c, shown at the right of Fig. 1, have k_V's with the greatest variation with temperature, while compounds which are not superconductors or have low transition temperatures have temperature-independent Knight shifts. Pure V metal is included in Fig. 1 for comparison.

The Knight shifts of the minority constituents of these compounds are shown in the lower part of Fig. 1. Some previous measurements[4] on V_3Ga are shown by the open squares. While the NMR of the minority constituents of only a few compounds can be studied, all of these show negative Knight shifts (i.e., diamagnetic shifts) with the exception of V_3Co which shows a temperature-independent $k_{Co} = +0.7\%$. Note that k_{Ga} in V_3Ga exhibits a strong temperature dependence.

The V-Ga intermetallic system forms the β-W structure in a wide range of composition (the V/Ga ratio is variable between the limits of 2 and 4). In the case $V/Ga \neq 3$, the excess V or Ga occupies Ga or V positions, respectively. The composition $V/Ga = 3$ is analogous to a pure metal, while the case $V/Ga \neq 3$ is somewhat analogous to a substitutional alloy and may be termed a mixed intermetallic compound. In this range the superconducting transition temperature[5] T_c has

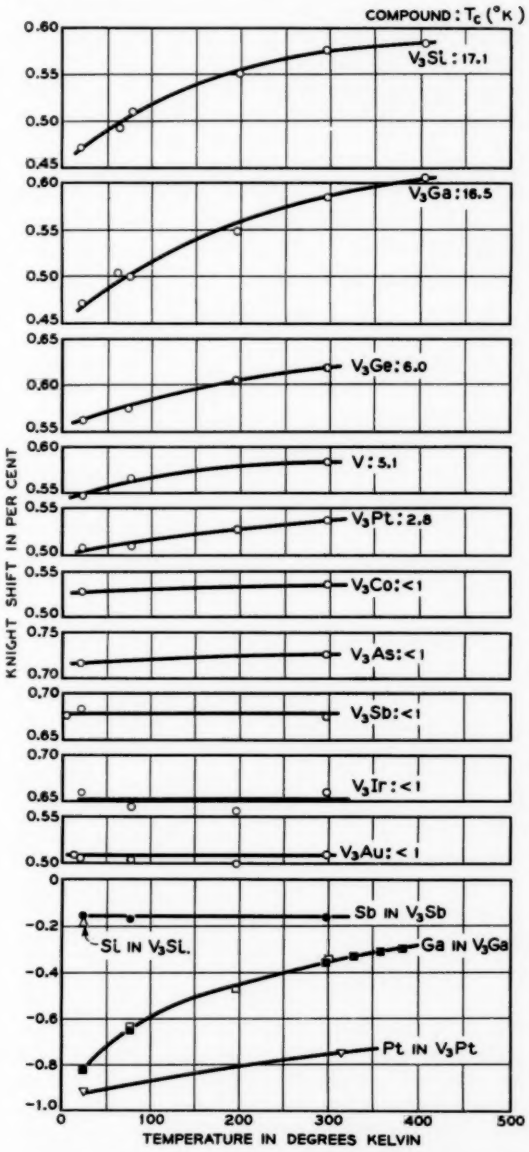

FIG. 1. Knight shifts k_V and k_X in % vs temperature for the intermetallic compounds V_3X. The values of k_V appear in the upper portion of the figure and are all positive; the k_X appear in the lower portion and are all negative. (k_{Co} is an exception to this rule and is discussed in the text.) Note that the scales for k_V and k_X are different. The superconducting transition temperatures T_c are tabulated at the right.

a maximum of 16.5°K at V/Ga = 3, decreasing monotonically on the Ga-rich side to 7.6°K at V/Ga = 2 and on the V-rich side to 10.1°K at V/Ga = 4. Figure 2 shows k_V and k_{Ga} for this system plotted as functions of T_c, with the temperature at which measurements were made as a parameter. Note that the compounds with the highest T_c show the greatest temperature dependence in both k_V and k_{Ga} (see Fig. 1).

The linewidths δH of the vanadium resonances measured between absorption derivative extrema are shown as a function of temperature in Fig. 3. At room temperature all V linewidths shown were approximately equal to that of V metal. The δH for all these compounds was independent of the magnetic field and essentially independent of temperature except in the case of V_3Ga. In addition, the V linewidth was found to be 15 oe in V_3Au and 50 oe in V_3Sb at 300°K and 14 koe. These linewidths, however, decreased at lower fields, showing that some anisotropic magnetic interaction not important for the other compounds determines δH for V_3Au and V_3Sb. The Ga^{71} linewidth in V_3Ga increased from 9 oe at 295°K to 40 oe at 20°K, while δH for Pt and Sb remained constant.

Most of the V resonance lines showed some quadrupole splitting.[6] The symmetry of the V site allows an axial electric field gradient. In Fig. 4 we have plotted e^2qQ against T_c of the compound involved. We note that there is a general trend for the superconductors having higher T_c to also have larger e^2qQ.

It is interesting to note that the magnitude of k_V for all these compounds except V_3As and V_3Sb is within 7% of k_V for V at room temperature. The fact that the temperature dependence of k_V in the case of V_3Si and V_3Ga is five times larger than the variation between the various compounds suggests that the temperature dependence of k_V be associated with the superconducting properties rather than with the particular kind of atom occupying the minority site. In the V-Ga system it

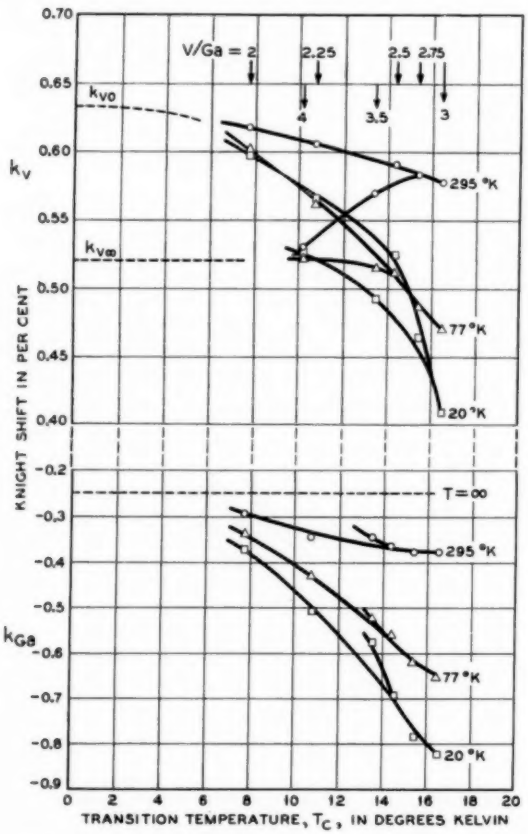

FIG. 2. Knight shift k vs T_c for the mixed intermetallic compound system V-Ga with the β-W structure. The value for k_V and k_{Ga} appear in the upper and lower portions of the figure, respectively. Note that the ordinate scales are different and that $k_V > 0$ while $k_{Ga} < 0$. The T_c corresponding to a given V/Ga ratio is given at the top. The extrapolated limits k_{V0} and $k_{V\infty}$ corresponding to V/Ga = 0 and ∞, respectively, refer to the expected local susceptibility difference between solvent and solute atoms as computed by J. Friedel, Phil. Mag. 43, 153 (1952). The extrapolated value for k_{Ga} at $T=\infty$ indicates that in the β-W structure it is intrinsically negative.

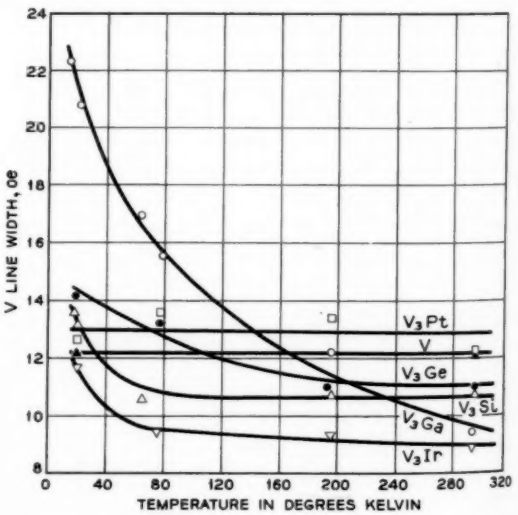

FIG. 3. Linewidth of V resonance as a function of temperature.

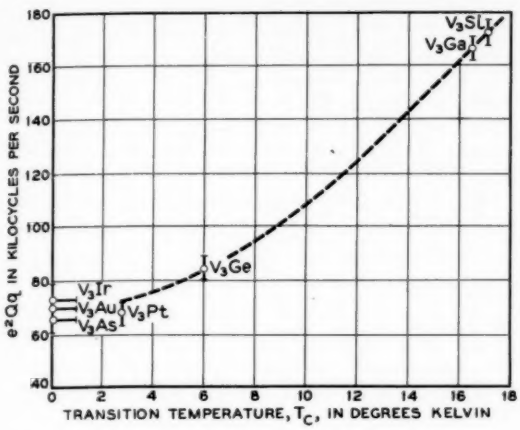

FIG. 4. Quadrupole interactions of V in β-W intermetallic compounds vs superconducting transition temperature. For V in V_3Co an anomalously large e^2qQ of 1.0 Mc/sec is observed (see discussion in the text).

is clear that the temperature dependence of k_V and k_{Ga} does not arise directly from the change in composition but is a function of the superconducting transition temperature. That is to say, those interactions which are responsible for the magnitude of T_c are also responsible for the temperature dependence of k.

At room temperature, linewidths of the V resonances are all considerably larger than the direct nuclear dipole-dipole interaction would allow. The two high-temperature superconductors show distinctly different temperature dependence: the V linewidth in V_3Ga suffers a large increase at low temperatures while that in V_3Si does not. We believe that the temperature-dependent and temperature-independent parts of δH arise, respectively, from the isotropic and anisotropic parts of the indirect nuclear exchange interaction[7] via the conduction electrons. Line broadening arises from the isotropic part only for dissimilar moments, but the anisotropic part produces broadening from the interaction between all moments. (All Ga nuclei have relatively large magnetic moments while less than 5% of the Si nuclei have any moment at all). The behavior of the Ga and Si resonances in these compounds is also consistent with this interpretation. However, δH for none of the low-temperature superconductors or nonsuperconductors has an appreciable temperature dependence, regardless of the abundance and size of the nuclear moments at the minority site. Those electrons which are directly responsible for the Knight shift and the isotropic indirect exchange interaction are the s electrons. The absence of the temperature dependence of δH in that case where isotropic indirect exchange is not possible suggests that we associate the temperature-dependent effects in the superconductors with the s electrons.

In the absence of extraordinary shielding by conduction electrons, one would expect from the behavior of the noncubic pure metals that the quadrupole interaction at the V site would be several Mc/sec. The smallness of e^2qQ is thus puzzling, but perhaps the most surprising feature is that the magnitude of e^2qQ should increase with increasing T_c. Thus, it appears that those interactions which are responsible for the magnitude of T_c decrease the shielding of the quadrupole interaction at the V site.

The facts that all the k_V are approximately the same and that all V atoms have only other V atoms as nearest neighbors, suggest a model for these compounds in which the V atoms form a sublattice having electronic properties not unlike V metal. The atoms at the minority site have only V neighbors and thus do not form such a sublattice. The fact that the Knight shifts are negative at the X sites in V_3X, whereas they are positive in each pure X metal, suggests that due to the formation of the intermetallic bond, the polarization of the available outer s electrons is reversed. This is in contrast to isolated solute metal atoms in a solvent metal lattice where the observed k_{solute} is always positive. The positive k_{Co} in V_3Co indicates that perhaps V_3Co is more like the latter "alloy" case than the former "intermetallic compound" case in that the overlap of the like s- and d-band electrons of Co and V negate the bonding effects, although V_3Co in all its other physical properties behaves like an intermetallic compound. It is interesting to note in this respect that V_3Co has an anomalously large V quadrupole interaction relative to the other V_3X compounds.

To our knowledge this is the first time that correlations have been observed between T_c and local atomic properties. These correlations exist at temperatures well above the superconducting region.

We wish to thank E. Corenzwit for making many of the compounds studied in these experiments, Mrs. V. Compton for x-ray analysis verifying their β-wolfram structure, and A. M. Clogston and H. Suhl for many instructive discussions.

[1]C. H. Townes, C. Herring, and W. D. Knight, Phys.

Rev. 77, 852, (1950), W. D. Knight, in Solid-State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1956), Vol. 2, p. 93.

[3]In addition to these compounds, V_3Sn was also prepared, but it was not possible to make it without excess metal dispersed in the lattice, although this was possible for all other samples. The samples were prepared by direct combination of the metals in an arc furnace.

[3]S. Geller, Acta Cryst. 9, 885 (1956). The β-wolfram structure has space group $Pm3n$. The point group at the V site is $\bar{4}2m$ and at the X site $m3$.

[4]R. G. Shulman, B. J. Wyluda, and B. T. Matthias, Phys. Rev. Letters 1, 278 (1958).

[5]T_c for these mixed intermetallic compounds is reported here for the first time.

[6]M. H. Cohen and F. Reif, in Solid-State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1957), Vol. 5.

[7]M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954).

MÖSSBAUER EFFECT IN Fe^{57} AT VERY LOW TEMPERATURES

J. G. Dash, R. D. Taylor, P. P. Craig, D. E. Nagle, D. R. F. Cochran, and W. E. Keller
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received July 14, 1960)

The recoil-free gamma-ray transition between the 14.4-kev state and the ground state of Fe^{57} has been used to study the hyperfine splittings of these levels in several crystals.[1-4] The studies of the magnetic hyperfine spectra in metallic iron[1] have shown the gamma ray to be split into six components characterized by a ground-state level separation $g_0 = 1.90 \times 10^{-7}$ ev, and a first excited state splitting of 1.07×10^{-7} ev. The separation of the ground-state levels is equivalent to a temperature of 2.2×10^{-3} °K, which has suggested to us the possibility of observing the effects on the Mössbauer fluorescence pattern that can be produced by cooling a resonant absorber to very low temperatures. In thermal equilibrium the population ratio of the Zeeman levels of the ground state of absorbing Fe^{57} nuclei ($I = 1/2$) varies with temperature as $\exp(-g_0/kT)$. Such a population asymmetry would produce an increased absorption coefficient for a transition arising from the $m = -1/2$ Zeeman level relative to transitions in which the m value is reversed in sign. Examination of these absorption coefficients could be made by measuring the transmission of resonant gamma rays from a source at room temperature, by the technique of Doppler-shifting the source relative to the cold absorber.

Larger asymmetries are produced by cooling the source instead of the absorber, since the Co^{57} parent has a magnetic moment about 50 times larger than that of the ground state of Fe^{57}. Success of this scheme depends upon the reasonable expectation that the decay of the Co^{57} through the second and first excited states of Fe^{57} proceeds so rapidly that the Zeeman level populations are unchanged.[5] The process is complicated and the over-all effect reduced, however, by the partial mixing of levels according to the character of the decay. Calculations based upon the assumptions that the electron capture process of Co^{57} is an allowed transition and that the decay from the second to the first excited state of Fe^{57} is either $M1$ or $E2$[6] indicated a partial preservation of the Co^{57} population ratios. These calculations predicted a larger over-all effect than the simpler scheme of cooling an absorber; accordingly the experiment was performed with a cold source and an absorber at room temperature.

A source of approximately 10 millicuries of Co^{57} in natural Fe metal was prepared by the usual technique of plating followed by thermal diffusion,[7] and installed in an adiabatic demagnetization cryostat fitted with thin windows of Mylar plastic and Be metal. The source foil was soldered to the bottom of a capsule containing about 0.1 mole of chromium potassium alum crystals in toluene, and this capsule was thermally attached to a liquid He^3 reservoir by a Pb "heat switch". The 14.4-kev radiation was filtered by a resonant absorber of approximately 2 mg/cm^2 Fe^{57} plated on 0.1 mil nickel foil. The absorber was situated outside the cryostat in a frame which could be oscillated linearly at various speeds by means of a synchronous motor, gear train, and cable system. Transmitted intensities were measured using a 1-mm thick NaI(Tl) crystal and photomultiplier, single-channel pulse-height analyzer, and digital printer. Periods of oscillation were varied from 3 seconds to 5 minutes, and intensities were measured alternately in each direction over the same length of absorber travel. Special care was exercised

in order to avoid transmission of vibration to the absorber and to the source. It proved necessary to conduct the measurements with the pumps stopped and the liquid He^4 at atmospheric pressure.

The Doppler pattern at room temperature showed a central absorption "peak" of 0.6 mm/sec width at half height, corresponding to a "thick" foil having a corrected resonant absorption of approximately 60%. The absorption peaks at ±2.2 mm/sec Doppler velocity had a comparable width, and a magnitude of 24% relative to the transmission at high velocities; these satellite lines were judged most suitable for the low-temperature study.

The Doppler pattern was also measured with the source at 4°K; the principal change from the spectrum at room temperature was a shift of 0.1 mm/sec toward positive velocities (positive velocity taken as increasing separation between source and absorber), in qualitative agreement with previous investigations.[8,9] The intensity asymmetry between the satellite lines was less than 0.6%, as expected. The satellite lines appeared at -2.1 and +2.3 mm/sec. The character of the Doppler drive system and our expectation that temperatures would change rapidly after demagnetization led us to choose a compromise speed, 2.21 mm/sec, for most of the work at low temperatures. This choice introduced only a 15% loss in sensitivity due to the displacement from the peaks. The chosen velocity was not precisely equally displaced from each line; this led to a basic difference between the intensities at (+) and (-) velocities, giving a ratio $I_-/I_+ = 0.99$. This asymmetry at 4°K formed the baseline for comparison of the peaks at lower temperatures.

The absorber and photomultiplier were removed, a field of 11 kilogauss applied to the salt, and the temperature reduced to 0.5°K by pumping on the liquid He^3 chamber. Thermal equilibrium was obtained in about one hour; the magnetic field was reduced to zero and the magnet moved away, the cryostat pumps were turned off, the liquid He^4 was warmed to its boiling point, and the foil and photomultiplier were replaced. Carbon thermometers attached to the source gave a qualitative measure of the temperature, which was about 0.2°K within 2 minutes after demagnetization. Measurements of the satellite lines were begun at this time; a pronounced intensity asymmetry $I_-/I_+ = 1.03$ was found at the compromise velocity. The ratio increased to about 1.04 during the next two minutes, while the thermometers indicated

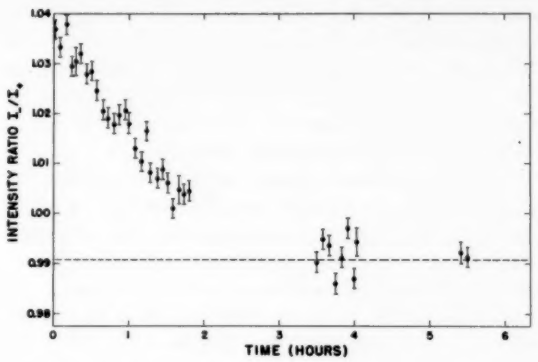

FIG. 1. Ratio of intensities transmitted at ±2.21 mm/sec during warmup.

continued cooling of the sample. The temperature increased thereafter, reaching a stable value near 1.4°K in six hours, and during this time, the intensity ratio decreased toward 0.99 as shown in Fig. 1. A background correction would increase all ratios by 0.6%. Checks of the intensities at the central absorption peak and at high velocities showed no appreciable changes during the warm-up period.

The observed variation of the absorption pattern with temperature demonstrates that the nuclear spin level population asymmetries are partially preserved through the two nuclear transitions. Uncertainty in the temperature makes inadvisable a quantitative analysis of these data; however, the observed magnitude of the asymmetry as a function of temperature agrees approximately with the preliminary calculation.

It is possible to deduce the direction of the hyperfine magnetic field at Co^{57} nuclei in Fe metal. The ratio I_-/I_+ increases above unity at lower temperatures, implying that the higher energy emission lines are becoming more intense, and the higher energy levels of the first excited state of Fe^{57} are increasingly populated. According to Hanna et al.,[1] this state has an inverted order; i.e., the -3/2 level is highest. Since any decay scheme requires that these levels are preferentially connected to the negative m-value levels of the parent states, the Co^{57} sublevels having negative m values have larger populations. These larger populations must arise from the influence of the Boltzmann factor. Therefore, the lower energy levels have negative m values. One is confident from shell-model considerations that the g value of the Co^{57} nucleus is positive. Since the level scheme is in the same order as that for the ground state of Fe^{57}, which

also has a positive g value, we conclude that the hyperfine field at the Co^{57} nuclei is parallel to that at the Fe. This has been found to be negative with respect to the ferromagnetic domain magnetization[10]; hence the field at the Co^{57} is also opposite to the domain field.

It is amusing to consider that the net maximum asymmetry ratio of 1.05 implies that the sublevels of the 14.4-kev state have populations corresponding to a small negative temperature, approximately -5×10^{-3} °K. The negative spin temperature results from the inversion of the sublevel ordering, and its absolute value is a factor of about 20 lower than the temperature of the lattice and the conduction electrons.

Application of the effect as a low-temperature thermometer appears to be a distinct possibility. We are presently conducting a more quantitative study of the temperature dependence; a detailed report on the work will be submitted.

We wish to thank W. Visscher for several important calculations and helpful discussions.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]S. S. Hanna, J. Heberle, C. Littlejohn, G. J. Perlow, R. S. Preston, and D. H. Vincent, Phys. Rev. Letters 4, 177 (1960).

[2]G. K. Wertheim, Phys. Rev. Letters 4, 403 (1960).

[3]O. C. Kistner and A. W. Sunyar, Phys. Rev. Letters 4, 412 (1960).

[4]Proceedings of the Allerton Park Conference on Mössbauer Effect, University of Illinois, Urbana, Illinois, June 5-7, 1960 (unpublished), Sec. 3.

[5]A. C. Gossard and A. M. Portis, Phys. Rev. Letters 3, 164 (1959); M. J. Steenland and H. A. Tolhoek, in Progress in Low Temperature Physics, edited by C. S. Gorter (North Holland Publishing Compnay, Amsterdam, 1957), Vol. 2, p. 326.

[6]W. M. Visscher (private communication).

[7]R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters 3, 554 (1959).

[8]R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters 4, 274 (1960).

[9]B. D. Josephson, Phys. Rev. Letters 4, 341 (1960).

[10]S. S. Hanna, J. Heberle, G. J. Perlow, R. S. Preston, and D. H. Vincent, Phys. Rev. Letters 4, 513 (1960).

SOURCE OF POLARIZED DEUTERONS AND THE VERIFICATION OF ALIGNMENT WITH THE $T(d,n)He^4$ REACTION

E. Baumgartner, L. Brown, P. Huber, H. Rudin, and H. R. Striebel
Department of Physics, University of Basel, Basel, Switzerland
(Received July 22, 1960)

Interest has developed in recent years in the construction of a source of polarized particles, and several suggestions have been made for its accomplishment.[1] A proposal by Galonsky, Willard, and Welton[2] for using the $T(d,n)He^4$ reaction simplifies the detection of the polarized beam. We have constructed an ion source for polarized deuterons and have confirmed the alignment with this reaction.

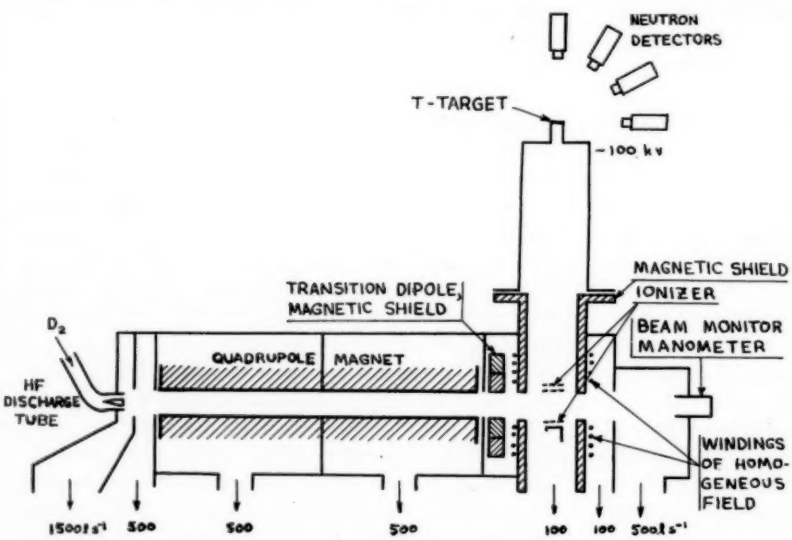

FIG. 1. Schematic diagram of the source.

The course of the particles through the device is shown in Fig. 1. Atoms of deuterium are allowed to diffuse from a high-frequency discharge tube through a region of differential pumping into a strong magnetic quadrupole field. This field is generated by permanent magnets attached to four parallel pole shoes 90 cm long, and has a value of 11 000 gauss at the surface. In this field atoms with strong-field quantum num-

bers $m_e = 1/2$ and $m_d = +1, 0, -1$ are confined to a circular cylindrical region of 10-mm diameter defined by the pole shoes; the other atoms diverge from the axis. The beam so formed passes into a orienting homogeneous field weak enough to allow strong coupling of electron and nucleus. A small part of it is ionized there by electron bombardment. Deuterons so produced have spin populations of 4/3, 4/3, 1/3 for $m_d = +1, 0, -1$, respectively. The resulting ions are accelerated through 100 kv onto a thick tritium target to produce the desired nuclear reaction. The portion of the atomic beam not ionized finally passes into a beam-receiving compartment where it is pumped away to reduce back diffusion.

The intensity of the atomic beam is measured by placing an ionization manometer in its path; the observed instrument reading is corrected for pressure buildup in the glass cylinder, the ionization cross section of hydrogen, and the divergence of the beam outside the quadrupole field. The measured beam intensity is 8×10^{15} sec^{-1} and the beam pressure at the point of ionization is 1×10^{-8} mm Hg. This manometer serves as the atomic beam monitor during operation.

On leaving the quadrupole field the atoms pass through a homogeneous transition field of about 60 gauss generated by an electromagnet, the ring yoke of which acts as the first magnetic shield between ionization volume and quadrupole magnet. They then pass through an opening in the iron cylinder enclosing the electron gun. Here a magnetic field of 1 gauss or more is maintained by external current windings in a direction perpendicular to the atomic beam axis and parallel to that of the ionic beam. This field is superimposed on the somewhat stronger field formed by the heating current of the tungsten cathode and a compensation winding, forming together a field similar to one produced by Helmholtz coils.

The ions are accelerated without magnetic deflection onto the target. Unpolarized deuterons are obtained by admitting deuterium gas into the ionization compartment. The spin axis is parallel to the ion beam; angular measurements are made relative to this direction. We measured the ratio of reaction cross sections $\sigma(0)/\sigma(\theta)$ for three angles simultaneously with four scintillation detectors by comparing the neutron counting rates produced by deuterons from the atomic beam and from molecular gas admission. A convenient parameter, G, is the population ratio of spin state 0 to the sum of the states +1 and -1. Figure 2 shows the measured distribution and the curves predicted for $G = 4/5$ and for the average value $G = 0.706(\pm 0.014)$. The average value is smaller than 4/5, as one might expect, since the field in the ionization volume is not exactly parallel; we think other sources of depolarization are of less importance. The magnetic shielding and the field from the cathode prevent a controlled change of the direction of the field in the ionizing region; nevertheless, investigations with two neutron detectors showed values of $\sigma(0°)/\sigma(90°)$ about unity, if part of the homogeneous field decreased below 1 gauss. Corrections for the different distributions in the laboratory co-

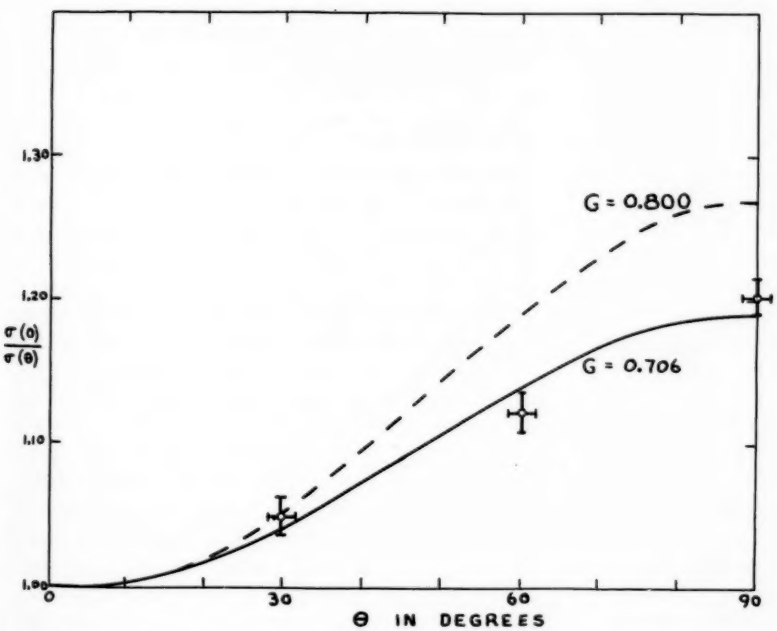

FIG. 2. The ratio of the differential reaction cross section at 0° to the cross section at angle θ is plotted for three measured values. The dashed curve shows the predicted distribution for the maximum possible value of G obtained from the three measured points.

ordinate system for the 100-kev and 50-kev (unpolarized molecular ions) particles are negligible when compared with the resolving power of the neutron detectors.

The current of the polarized beam can be calculated from a knowledge of counting rates, total ion current, partial pressure of deuterium admitted for the unpolarized ions, and the geometry of the electron gun. The current so calculated is 0.01 μa. Typical neutron counting rates are as follows: atomic beam ionized, 700 min^{-1}; residual gas ionized with normal gas admission but no atomic beam, 170 min^{-1}; residual gas ionized without gas admission, 130 min^{-1}; no ionization but accelerating voltage present, 50 min^{-1}.

Investigations of the residual gas ions show the source in its present form is unsuitable for the generation of polarized protons, since about 5 times as many protons originate in the residual gas as in the atomic beam. This is attributed for the most part to the use of rubber seals and oil diffusion pumps; furthermore, no special care has been taken to reduce the residual gas pressure.

---

[1]A detailed description of the source with an extensive bibliography will soon appear in Helvetica Physica Acta.

[2]A. Galonsky, H. B. Willard, and T. A. Welton, Phys. Rev. Letters 2, 349 (1959).

NEGATIVE PION-PROTON ELASTIC SCATTERING AT 600 TO 750 Mev*

John I. Shonle
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received July 22, 1960)

The total π^--p cross section shows two well-defined peaks, one at 600 Mev and one at 890 Mev (laboratory-system kinetic energy).[1] Peierls[2] has assigned $D_{3/2}$ and $F_{5/2}$, respectively, for the orbital and total angular momentum states based on the photoproduction angular distributions[3] and polarizations of the recoil protons.[4] Landovitz and Marshall[5] suggest that $P_{3/2}$ and $D_{3/2}$ or $D_{5/2}$ assignments are also consistent with the data. Previous π^--p elastic scattering experiments have been made at 425,[6] 460,[7] 600,[7] 770,[7] 810,[8] 925,[9] and 950[10] Mev. These experiments have not led to any definite conclusions, partly because of large energy spreads and low statistics.

This experiment was conceived to try to establish the angular momentum at the peaks from elastic scattering on hydrogen. Negative pions at 610±20, 655±20, and 750±20 Mev were passed through a 30-inch propane bubble chamber operated in a 13-kgauss field.[11] The pions were focused, deflected, and collimated to give a momentum spread of ±1.5%. The energy spread quoted above comes from energy loss in the chamber. The mean beam momentum was checked by wire orbiting, by measuring the curvature of beam tracks in the bubble chamber, and from kinematics of elastic scattering events with stopping protons. All three methods gave consistent results. Twenty percent of the film was scanned twice. All events were measured on digitized microscopes (most of them on a "Franckenstein") and the data reduced on an IBM 650. A kinematics program gave the events a χ^2 test for elasticity using configuration-dependent errors. Good agreement with the expected χ^2 distribution was found. About 40% of the measured events were elastic.

Tracks entering the scanning region were counted in 4% of the pictures. Corrections were made based on calculated muon contaminations (11.5%) and measured electron contaminations (about 3%), and for interactions reducing track length. The resulting track lengths were checked by counting interactions and checking with the π^--p [1] and π^--C [12] total cross sections.

The numbers of elastic scattering events found were 539, 1159, and 1008 at 610, 655, and 750 Mev, respectively. Analysis of about 20% more events is still in process, and corrections were made for these assuming they were randomly distributed. Corrections were made for scanning efficiency and azimuthal bias, but not as a function of scattering angle. A correction was made for carbon contamination (about 7%) by using the behavior of the nonelastic tail of the observed χ^2 distribution. For the total elastic scattering cross sections, corrections for small-angle scattering events which were missed were made

by extrapolating the angular distributions to 0 deg. The resulting total elastic scattering cross sections are 16.6 ± 2.2 mb, 16.1 ± 1.6 mb, and 14.4 ± 1.3 mb, at 610, 655, and 750 Mev, respectively. The errors in the cross sections are thought to be about standard deviations, and are not strongly correlated from one energy to another. These results are compared with the results of other workers in Fig. 1. If the latest total cross sections[1] are valid, then the results at 810 and 950 Mev should possibly be scaled to where the crosses indicate. Within the errors shown, almost any energy dependence from one with peaks at 600 and 890 Mev to one which increases linearly may be concluded.

The angular distributions are shown in Fig. 2. The crosses indicate the expected forward scattering calculated from the optical theorem and dispersion relations.[13] The extrapolations of the observed angular distributions to 0 deg are compatible with these points if the cross sections are rising at small angles as a diffraction pattern with reasonable values of the radius. Fits of cosine power series were made to the data. In each case the intervals $0.9 \leq \cos\theta^* \leq 1.0$ were not considered, and only statistical errors were used, since other errors affect only the over-all normalization. Fits were made with the data divided into intervals of $\Delta(\cos\theta^*) = 0.05$ (38 points) and $\Delta(\cos\theta^*) = 0.10$ (19 points). The two fits agreed within errors with each other. The 19-point fits are given in Table I. The values of χ^2 reached plateaus at the powers of cosine shown. In all cases the coefficient of the next higher power of cosine was zero within the errors. The errors quoted have the normalization errors folded back in.

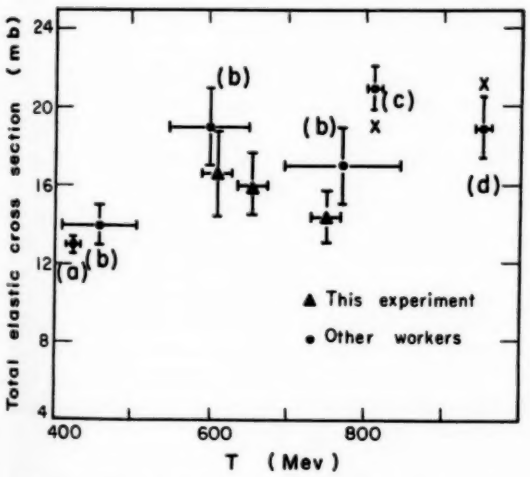

FIG. 1. The total elastic scattering cross section is given as a function of the pion kinetic energy (laboratory system). The crosses show suggested changes in the elastic scattering data based on the latest total cross-section data. (a) Reference 6; (b) reference 7; (c) reference 8; (d) reference 10.

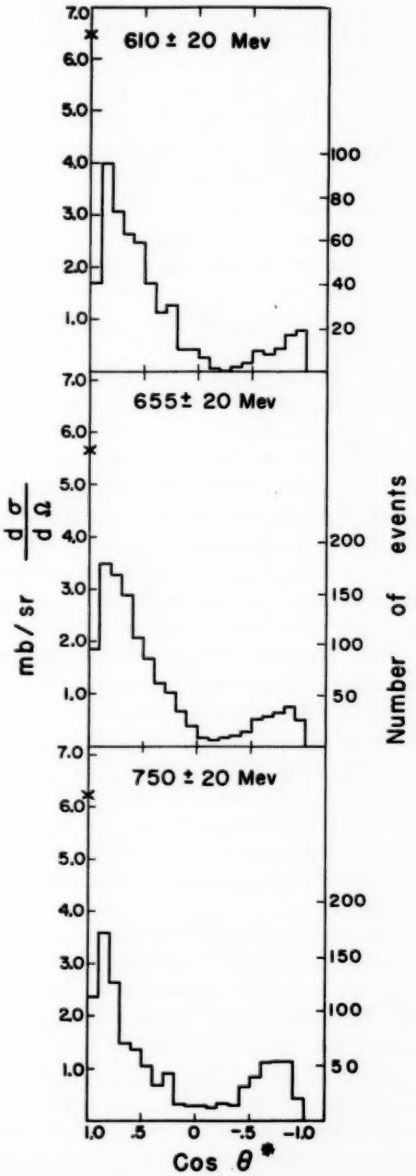

FIG. 2. The elastic scattering angular distributions are given as a function of the cosine of the angle in the center-of-momentum system. The crosses are the predicted forward scattering cross sections.

Table I. Coefficients of the various powers of $\cos\theta^*$ (in mb/sr) for the 19-point fits. The highest power and the value of χ^2 for the fit are given. There were $(18-n)$ degrees of freedom.

Energy (Mev)	n	χ^2	a_0	a_1	a_2	a_3	a_4	a_5
610	4	10.6	0.23 ± 0.04	1.73 ± 0.28	3.78 ± 0.61	0.27 ± 0.44	-1.23 ± 0.69	...
655	4	7.2	0.25 ± 0.04	1.56 ± 0.20	4.38 ± 0.50	0.17 ± 0.32	-2.54 ± 0.53	...
750	4	16.3	0.25 ± 0.04	0.81 ± 0.20	2.49 ± 0.48	-3.24 ± 1.02	0.52 ± 0.69	5.81 ± 1.22

The fact that a_2 and a_4 were found to be larger at 655 Mev than at 610 Mev should not be taken too seriously. It is assumed for what follows that a_2 is really at a maximum at 600 Mev. If the peaks in the cross section are resonances—that is, the real part of the phase shift goes through 90 deg—then the size of σ_{elas} compared to $\frac{1}{2}\pi(2J+1)\lambda^2|e^{2i\delta}-1|^2$ makes it likely that the first peak has $J \leq 3/2$ and the second $J \leq 5/2$. That the coefficient a_2 goes through a maximum at 600 Mev implies that the angular momentum state there is $J = 3/2$. The decrease in a_2 and a_3 and the increase in a_4 and a_5 all imply that the next peak should have $J = 5/2$. If it is assumed that both resonances go through +90 deg and that there is a reasonable energy dependence of the phase shifts, then even relative parity for the two resonances would lead to interference terms that would give negative contributions to a_2 and positive to a_4 around 600 Mev. It does not seem easy to reconcile such contributions with the observed coefficients. It would be difficult to explain a_2 and a_4 if one resonance goes through +90 deg and the other through -90 deg. Therefore it is most reasonable to assign odd relative parity to the resonances: $P_{3/2}$ and $D_{5/2}$, or $D_{3/2}$ and $F_{5/2}$. Thus the same possible angular momentum assignments are arrived at independently of the photoproduction data.

Several nonresonating states are needed to explain the observed angular distributions. Plausible sets of such other states have been constructed. The problem is underdetermined by the data at hand, since the phase shifts are complex. If some of these other states could be determined by other means, then the ambiguity of the orbital momentum of the resonances could possibly be eliminated. Because the nonresonant states are responsible for the negative value of a_4 around 600 Mev, its minimum is probably due to "accidental" cancellations. If the real part of the phase shift of the $J = 3/2$ wave is 90 deg at 600 Mev, then the phase must have a large imaginary part. This conclusion is necessary to account for the observed amount of inelastic scattering when it is kept in mind that there must be some nonresonant elastic scattering. Strong absorption is in keeping with Peierls' conjecture[2] as to the cause of this peak. It should be emphasized that this experiment does not prove that the peaks are true resonances. However, it would be more difficult to account for the data without that hypothesis.

Isospin has been ignored in this qualitative analysis. One difficulty with this analysis is that a_3 and a_5 seem to be changing faster than general considerations would indicate likely. Part of the apparent sudden change may be due to a lack of sensitivity to the fifth and sixth powers of cosine at the lower energies. The supposition of insensitivity is compatible with the results of Goodwin et al.,[6] who find a best fit with a sixth-degree polynomial at 425 Mev.

The support of Dr. Wilson Powell is gratefully acknowledged. Dr. Oreste Piccioni, Dr. William Fowler, and Dr. Robert Birge helped immensely with the early stages of this experiment. We would like to thank Dr. Edward Lofgren and the Bevatron crew for their help and skill in beam sharing, and Larry Oswald and the bubble chamber crew for chamber operations. Jack Hohenstein did much of the scanning and has been indispensible in helping to compile the data. Edith Goodwin and Herbert Holden also helped with the scanning. Howard White and his staff are thanked for the data reduction. The author has consulted frequently with Peter Newcomb.

---

*Work performed under auspices of the U. S. Atomic Energy Commission.

[1]J. C. Brisson, J. Detoef, P. Falk-Vairant, L. van Rossum, G. Valladas, and L. C. L. Yuan, Phys. Rev. Letters 3, 561 (1959); T. J. Devlin, B. C. Barish, W. N. Hess, V. Perez-Mendez, and J. Solomon, Phys.

Rev. Letters 4, 242 (1960).
[2]Ronald F. Peierls, Phys. Rev. Letters 1, 174 (1958); Phys. Rev. 118, 325 (1960).
[3]J. W. DeWire, H. E. Jackson, and R. Littauer, Phys. Rev. 110, 1208 (1958); P. C. Stein and K. C. Rogers, Phys. Rev. 110, 1209 (1958); M. Heinberg, W. M. McClelland, F. Turkot, W. M. Woodward, R. R. Wilson, and D. M. Zipoy, Phys. Rev. 110, 1211 (1958); J. I. Vette, Phys. Rev. 111, 622 (1958); H. H. Bingham and A. B. Clegg, Phys. Rev. 112, 2053 (1958); F. P. Dixon and R. L. Walker, Phys. Rev. Letters 1, 142 (1958); F. P. Dixon and R. L. Walker, Phys. Rev. Letters 1, 458 (1958); K. Berkelman and J. A. Waggoner, Phys. Rev. 117, 1364 (1960).
[4]P. C. Stein, Phys. Rev. Letters 2, 473 (1959).
[5]L. F. Landovitz and L. Marshall, Phys. Rev. Letters 3, 190 (1959).
[6]L. K. Goodwin, R. W. Kenney, and V. Perez-Mendez, Phys. Rev. Letters 3, 522 (1959).
[7]R. R. Crittenden, J. H. Scandrett, W. D. Shephard, W. D. Walker, and J. Ballam, Phys. Rev. Letters 2, 121 (1959).
[8]Lee Baggett, Jr., M. S. thesis, University of California Radiation Laboratory Report UCRL-8302, May, 1958 (unpublished).
[9]B. T. Feld, B. Maglić, and C. A. Diffey, Bull. Am. Phys. Soc. 4, 447 (1959).
[10]A. R. Erwin, Jr., and J. K. Kopp, Phys. Rev. 109, 1364 (1958).
[11]W. M. Powell, W. B. Fowler, and L. O. Oswald, Rev. Sci. Instr. 29, 874 (1958).
[12]J. W. Cronin, R. Cool, and A. Abashian, Phys. Rev. 107, 1121 (1957).
[13]James W. Cronin, Phys. Rev. 118, 824 (1960).

SENSITIVITY OF LOW-ENERGY PION-NUCLEON SCATTERING TO A PION-PION RESONANCE

S. C. Frautschi*
Department of Physics, University of California, Berkeley, California
(Received July 26, 1960)

Recently attention has been called to the possibility of explaining nucleon electromagnetic structure quantitatively by the introduction of a pion-pion resonance in the $T=1$, $J=1$ state.[1] Such a resonance should also affect pion-nucleon scattering. General expressions relating pion-nucleon to π-π scattering within the framework of double dispersion relations have been developed,[2,3] and some detailed applications have been made to low-energy π-N scattering.[4,5] In particular, it has been found that the $P_{3/2,3/2}$ π-N resonance is insensitive to π-π scattering.[4,5] However, the applications made to date have not been entirely satisfactory because the short-range π-N interaction is not well understood.

In this Letter we wish to point out that in π-N scattering the $J=1/2$, P states are extremely sensitive to the π-π resonance. The π-π contribution to these states near threshold is greater than the contribution to the $P_{T=3/2,J=3/2}$ state, roughly in the ratio[6]

$$P_{1/2,1/2}:P_{3/2,1/2}:P_{1/2,3/2}:P_{3/2,3/2}\sim 9:-4.5:-2:1.$$

We further wish to argue that the π-π resonance parameters which fit the nucleon electromagnetic structure make it difficult to obtain agreement with experiment in several π-N scattering states. This conclusion is especially strong in the $P_{1/2,1/2}$ state where, in addition to being large, the π-π contribution represents an attractive, long-range interaction which should lead to a large positive phase shift no matter what form the unknown short-range interaction may take.

Let us consider the $J=1/2$, P-state amplitude,

$$f_{J=1/2}^{P}(W)\equiv(q/\mu)^{-3}e^{i\delta}\sin\delta, \qquad (1)$$

where W and q are the total energy and the pion momentum in the center-of-mass system. In Table I we list the contribution to f at threshold

Table I. Comparison of Chew-Low theory, and Frazer-Fulco π-π resonance contribution, with π-N scattering experiments for the $J=1/2$, P amplitudes f [Eq. (1)] at threshold. The experimental numbers are from the 1958 CERN conference.[a]

	Chew-Low	π-π	Total	Exp.
$f_{1/2}^{P}(T=1/2)$	-0.14	+0.41	+0.27	-0.038 ±0.038
$f_{1/2}^{P}(T=3/2)$	-0.035	-0.21	-0.245	-0.044 ±0.005

[a]1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

from the well-known Chew-Low theory, and the additional contribution we obtain from π-π resonance scattering in the $N+\bar{N}\to 2\pi$ channel as calculated within the framework of double dispersion relations. Since the π-π contribution is unfamiliar and crucial to our argument, it is necessary to give some details concerning it. By the procedures of reference 5 it can be expressed as

$$ {}^{\pi\pi}f_{J=1/2}{}^{P}(M+\mu)\begin{pmatrix}T=1/2\\T=3/2\end{pmatrix}=\begin{pmatrix}-2\\1\end{pmatrix}\frac{\mu^3}{8\pi t_R(M+\mu)} \times\left[2\gamma_1\left(6+\frac{3\mu}{M}+\frac{8M\mu}{t_R}\right)+\gamma_2\left(16+\frac{24\mu}{M}+\frac{3t_R}{M^2}\right)\right], \tag{2}$$

where M and μ are the nucleon and pion masses, t_R is the energy of the π-π resonance, and the γ_i are related to the nucleon electromagnetic spectral functions[1] $g_i{}^V$ and the π-π form factor[1] F_π by

$$\gamma_1=-8\pi\left(\frac{g_1{}^V(t_R)}{e\,|F_\pi(t_R)|^2}\right)\left(1-\frac{4\mu^2}{t_R}\right)^{-3/2}, \tag{3}$$

$$\gamma_2=-8\pi\left(\frac{Mg_2{}^V(t_R)}{e\,|F_\pi(t_R)|^2}\right)\left(1-\frac{4\mu^2}{t_R}\right)^{-3/2} \tag{4}$$

We shall use the π-π resonance width and position ($t_R=11.5\mu^2$) with which Frazer and Fulco obtained the best results for nucleon electromagnetic structure. According to their work[1] the π-π resonance essentially contributes a factor $|F_\pi|^2$ to the "magnetic" spectral function $g_2{}^V$, and we can estimate $g_2{}^V/|F_\pi|^2$ in (4) quite accurately from the value which it takes in the absence of π-π scattering.[7,8] We estimate the "charge" term $g_1{}^V/|F_\pi|^2$ in the same way, although the estimate is less reliable in this case. With this procedure, 2/3 of the numerical value of Eq. (2) is given by the "magnetic" contribution so that our results are not especially sensitive to the uncertainty in $g_1{}^V$. We find that $\gamma_1=-6.9$ and $\gamma_2=-15.6$, which leads to the π-π results in Table I.

The π-π terms we have obtained at threshold appear to be much too large, and we proceed to strengthen this conclusion by some further consideration of the $P_{1/2,1/2}$ amplitude. When the quantities in Table I are calculated at energies above threshold, we find that the $P_{1/2,1/2}$ amplitude passes through resonance at less than 200-Mev kinetic energy, and that at this energy the attractive π-π contribution is at least 5 times the repulsive Chew-Low contribution. A closer analysis, of the type carried out in reference 5, can be made. Such analysis shows that the contributions we have considered in this Letter are dominated by the nearest singularities in the complex energy plane, which correspond to the longest range interactions in the problem, and that the terms we have not taken into account are of shorter range. Crude estimates of the short-range effects indicate they are attractive, but the main point is that even if other factors lead to a net "repulsive core," the attraction which we have found at longer ranges should still produce a large net positive phase shift. Thus the Frazer-Fulco resonance leads to a low-energy $P_{1/2,1/2}$ amplitude which is in disagreement with experiment.

We place somewhat less weight on the apparent disagreement in the $P_{3/2,1/2}$ state (Table I), because an attraction in the unknown short-range region could conceivably bring about agreement in this case.

Frazer and Fulco[1] found that the nucleon electromagnetic structure does not determine the π-π resonance parameters very precisely, so we must consider variations of the resonance width and energy. The quantities γ_1 and γ_2 are nearly constant over the range of parameters which are consistent with the electromagnetic data. It then follows from Eq. (2) that the π-π contribution varies $\sim t_R{}^{-1}$. The "range" of the π-π contribution also decreases $\sim t_R{}^{-1}$. Thus, although the discrepancy in the $P_{1/2,1/2}$ state persists at larger t_R, it becomes smaller and the argument concerning the "range of interaction" becomes weaker. It is also possible that the π-π channel is more complicated than has been imagined; for example, the existence of a 3π bound state[9] might force a re-examination of all the previous work on π-π effects.

Turning now to the recent work of Bowcock et al.[4] (B), we wish to argue that although these authors obtained a π-π contribution about 8 times smaller than that in Table I, their work actually strengthens our conclusion. There are two main factors which account for their smaller π-π term:

(*i*) The authors of B worked at $t_R=22.4\mu^2$, which is double our value and halves the π-π contribution as explained above.

(*ii*) The remaining factor of ~4 arises because the π-π resonance width was determined by different criteria in B than in Frazer and Fulco. In our work, following Frazer and Fulco,[1] the

π-π resonance width is determined by the anomalous magnetic moment. In B, no attempt is made to fit the magnetic moment. Instead, the resonance width is fit to the π-N S-wave energy dependence (the S-wave thresholds are taken from experiment, and then the more long-range-dependent curvature above threshold is calculated). This procedure leads to a much narrower resonance than the Frazer-Fulco treatment would give at $t_R = 22.4\mu^2$, and if the anomalous moment were calculated at $t_R = 22.4\mu^2$ with this narrow resonance it would come out much too large. When the fit to the energy dependence of the nucleon form factors is made, the narrow resonance width is responsible for the above-mentioned factor of 4 reduction of the $N+\overline{N} \to 2\pi$ amplitude in B.

Thus the moderate π-π contributions in B are obtained at the expense of failing to explain the nucleon magnetic moment; if the π-π resonance were fit to the magnetic moment the work in B indicates that the π-N S state as well as $J=1/2$, P state would disagree with experiment.

The early stages of this work were carried out in collaboration with Dr. J. D. Walecka. Discussions with Dr. G. F. Chew, Dr. S. Mandelstam, and Dr. W. R. Frazer were also helpful.

*Supported by the National Science Foundation. This paper was written while the author was a guest at the Latin American Summer School of Physics, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, June 27-August 7, 1960.

[1]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).

[2]S. W. MacDowell, Phys. Rev. 116, 774 (1959).

[3]W. R. Frazer and J. R. Fulco, Phys. Rev. (to be published).

[4]J. Bowcock, W. N. Cottingham, and D. Lurié (to be published). Hereafter called B.

[5]S. C. Frautschi and J. D. Walecka, Phys. Rev. (to be published).

[6]We give here the minimum ratio of $J=1/2$ to $J=3/2$ contributions, which holds at high π-π resonance energies t_R such as $t_R \sim 22\mu^2$; at lower t_R the preponderance of $P_{1/2,\,1/2}$ over $P_{3/2,\,3/2}$ increases. These ratios follow from Eq. (2) together with results from reference 5, and agree approximately with the static limit used in reference 4.

[7]G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. 110, 265 (1958).

[8]P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. 112, 642 (1958).

[9]G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

SINGLE PION PRODUCTION IN 0.96-Bev π^--p INTERACTIONS*

E. Pickup†
Division of Pure Physics, National Research Laboratory, Ottawa, Ontario

and

F. Ayer and E. O. Salant
Brookhaven National Laboratory, Upton, New York

(Received July 27, 1960)

Photographs of interactions in a hydrogen bubble chamber (field 13.5 kilogauss) irradiated by 0.96-Bev negative pions from the Cosmotron were kindly supplied by Professor Steinberger. We have measured and analyzed 415 events of the neutron reaction

$$\pi^- + p \to n + \pi^+ + \pi^-,$$

and 267 events of the proton reaction

$$\pi^- + p \to p + \pi^0 + \pi^-.$$

Experimental values quoted in this Letter all refer to the barycentric system.

Alles-Borelli, Bergia, Ferreira, and Waloscek[1] have shown, from measurements of other photographs in the same irradiation, evidence for the presence of a resonant state $T=J=3/2$; effects of this state have been calculated by Lindenbaum and Sternheimer.[2]

If this were the only resonant state involved, fast negative pions ("fast" designating a momentum ≥ 325 Mev/c) would originate mostly as recoils from $(n\pi^+)_{3/2}$ or $(p\pi^0)_{3/2}$; they would, accordingly, have the same angular distribution in both reactions, and (from Clebsch-Gordan coefficients) would be twice as numerous in the proton as in the neutron reaction. Figure 1 shows that the angular distributions are quite different, and the observed ratio of proton to neutron events with fast π^- is 0.9 ± 0.2. It follows then, that at

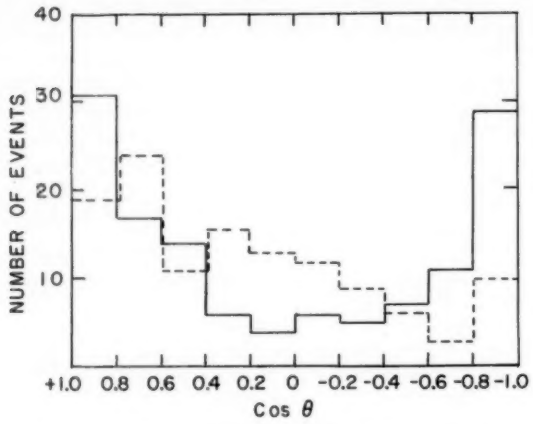

FIG. 1. Angular distributions of fast π^- particles. θ = barycentric angle between direction of produced and of incident pion. Solid line refers to neutron reaction, broken line to proton reaction.

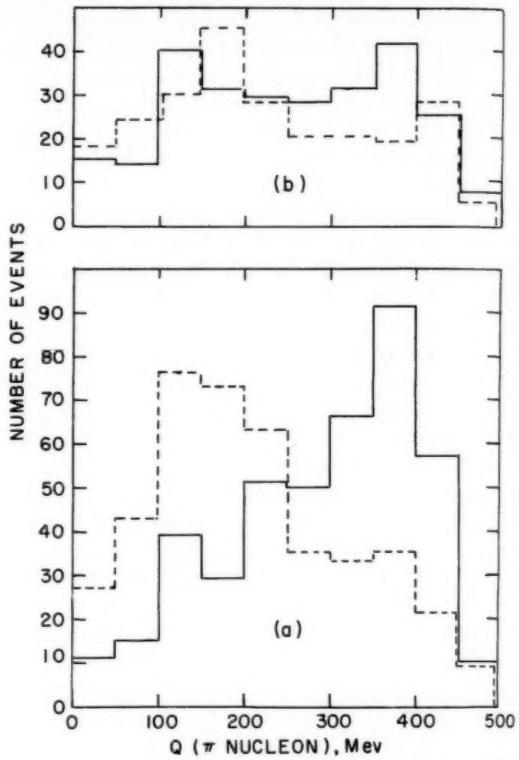

FIG. 2. Pion-nucleon Q values. (a) Solid line ($n\pi^+$), broken line ($n\pi^-$). (b) Solid line ($p\pi^-$), broken line ($p\pi^0$).

least one state, in addition to the 3/2, is involved (and interferes destructively with the 3/2 state).

Lindenbaum and Sternheimer have recently extended their calculations of branching ratios and momentum spectra to include contributions from the $T=1/2$ state as well as from the $T=3/2$ state.[3]

Q values for the different nucleon-pion combinations have been computed from the observed momenta, Fig. 2. The distribution for $(n\pi^-)$ shows a strong peak at 150 Mev, attributable to the $T=J=3/2$ state; the distribution for $(n\pi^+)$ shows at least as marked a peak at about 400 Mev, which can be interpreted as the effect of the lower $T=1/2$ state ($Q=430$ Mev). The $(p\pi^-)$ and $(p\pi^0)$ Q-value distributions show the same peaks, but much less marked.

If only the 1/2 and 3/2 isobaric states are involved in the neutron reaction, fast π^- particles would be practically all recoils in the production of $(n\pi^+)_{3/2}$. The approximate cosine-squared distribution, solid line of Fig. 1, means, in that case, that the state of relative angular momentum of $(n\pi^+)_{3/2}$ and π^- is a P state (unless there is an accidental mixture of angular momentum states). The angular distribution of positive pions in the rest system of $(n\pi^+)$, restricted to events with fast π^-, shows approximately isotropic decay of $(n\pi^+)_{3/2}$ in its own rest system. An Adair spin analysis[4] of these events rules out $J=1/2$ and is quite consistent with $J=3/2$.

Neutrons show approximately fore and aft symmetry in the barycentric system, whereas twice as many protons move backward as forward; neutron and proton momentum spectra differ as well. The backward protons give rise to a peak at ~100 Mev in the laboratory kinetic energy spectrum. The low-energy laboratory peak has been interpreted by Bonsignori and Selleri[5] and by Derado[6] as evidence for strong pion-pion interaction.

Figure 3 shows distributions of Q values for pion-pion combinations in each reaction, with backward-nucleon events and forward-nucleon events plotted separately. No evidence of peaking is seen for $(\pi^-\pi^+)$, either for forward or backward neutrons, nor for $(\pi^-\pi^0)$ for forward protons. However, the Q distribution for $(\pi^-\pi^0)$ in events with backward protons shows a distinct peak (at least as well marked as the proton-pion peaks), at about 325 Mev, suggestive of a 600-

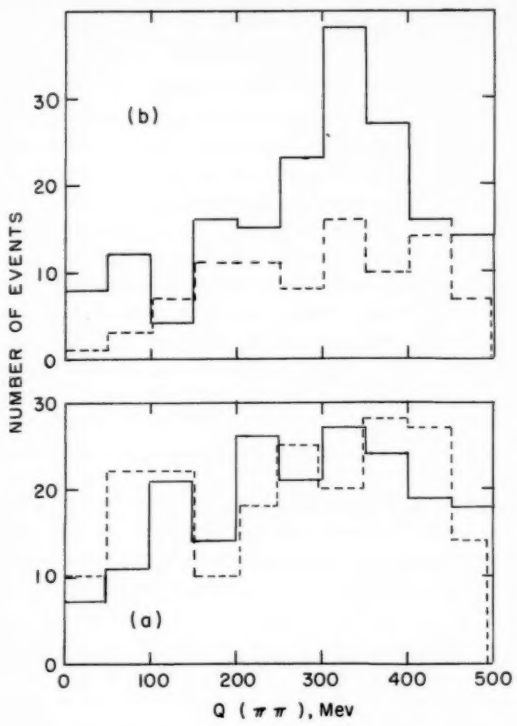

FIG. 3. Pion-pion Q values. (a) $(\pi^-\pi^+)$. (b) $(\pi^-\pi^0)$. Solid lines are for backward-nucleon events, broken lines for forward-nucleon events.

Mev resonant "di-pion" state, $T=1$ or 2. It will be interesting to see what happens to this peak in production by incident pions of different energies, now being analyzed.

Assuming that the Q peak really means a $T=1$ or 2 state, it may be noted that the observed ratio, 1.19 ± 0.14, of fast to slow π^- in the proton reaction does not lie within the limits of the ratio, 0.44-0 for $T=1$ and 16-4 for $T=2$, computed by Landovitz and Marshall,[7] so that one of these states alone would be inadequate to describe the proton reaction.

We are grateful for the assistance of the scanning staff, and of Mr. J. E. Smith in programming.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Guest physicist at Brookhaven National Laboratory, Upton, New York.

[1]V. Alles-Borelli, S. Bergia, E. Perez Ferreira, and P. Waloschek, Nuovo cimento 14, 211 (1959).

[2]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 105, 1874 (1957).

[3]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. Letters 5, 24 (1960).

[4]R. K. Adair, Phys. Rev. 100, 1540 (1955).

[5]F. Bonsignori and F. Selleri, Nuovo cimento 15, 465 (1960).

[6]I. Derado, Nuovo cimento 15, 853 (1960).

[7]L. Landovitz and L. Marshall, Phys. Rev. Letters 4, 474 (1960).

GENERALIZED ISOBAR MODEL, AND THE PIONIC FORM FACTOR OF THE NUCLEON*

W. Selove
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
(Received July 22, 1960)

Chadwick et al.[1] have studied the inelastic proton spectrum from p-p collisions at 1 to 2 Bev, and have found that although some features of the results are in rough agreement with predictions of the isobar model theory of Lindenbaum and Sternheimer,[2] there are several discrepancies of particular interest.

This note is to point out that in a simple model in which the isobar is excited by means of a virtual pion, the deviations can be understood in a simple way in terms of the decreasing probability of isobar excitation for increasing four-momentum transfer between the colliding nucleons. This decrease is due to two effects—(a) the characteristic energy-denominator associated with the virtual pion, in the transition-amplitude matrix element, and (b) the effect of the "pionic form factor" of the nucleon. This latter term is defined in analogy with the electromagnetic form factor.

The isobar model predicts moderately well-defined peaks in the inelastic spectrum, corresponding to the various nucleon isobars. These isobars are in turn considered to correspond to the peaks in the pion-nucleon cross section; the first three correspond to isobar masses $M_1' = 1.23$ Bev, $M_2' = 1.52$ Bev, $M_3' = 1.67$ Bev. The following features of the experimental results

are noted:

(A) Although production of the lowest isobar is indicated strongly by the data, production of the higher isobars is not evident.

(B) The lowest isobar appears to be produced in this experiment at a mass M' somewhat lower than the expected value of 1.23 Bev.

(C) A low-energy hump in the c.m. proton energy distribution, associated with double-M_1' production, is expected but not observed.

In order to see how these results can be interpreted in the manner indicated, consider the diagrams in Fig. 1. The invariant 4-momentum transfer is given by

$$q^2 = (\Delta\vec{P})^2 - (\Delta W)^2 = (\vec{P} - \vec{P}_0)^2 - (W - W_0)^2$$
$$= (\Delta\vec{p})^2 - (\Delta w)^2 = (\vec{p} - \vec{p}_0)^2 - (w' - w_0)^2. \quad (1)$$

Also, for the excited nucleon, which is originally at rest, $P_0 = 0$ and $W_0 = M$, so that

$$M'^2 = W^2 - P^2 = (M + w_0 - w')^2 - (\vec{p}_0 - \vec{p}')^2. \quad (2)$$

In (2), $\vec{p}_0$ and w_0 are the momentum and total energy of the incoming nucleon, $\vec{p}'$ and w' similarly for the outgoing unexcited nucleon; M is the nucleon mass; M', P, and W refer to the outgoing isobar.

In a perturbation-theoretic approach (which although certainly not adequate may nevertheless indicate the general features of this process), the transition matrix element for production of the equivalent 2-body state of Fig. 1(c) will be proportional to

$$F_{00}(q^2)F_{n0}(q^2, M_n')(q^2 + \mu^2)^{-1}, \quad (3)$$

where μ is the pion mass, and the F's are pionic form factors for the nucleon. For isobar formation by a real pion, $q^2 = -\mu^2$, and the pole given by (3) goes over into a finite cross section in the usual way. For virtual pions, however, q^2 is always non-negative, and with increasing q^2 the first two factors in (3) may be expected to decrease (in analogy with the electromagnetic case), along with the last.

Under the experimental conditions used in the work of Chadwick et al., the third factor of (3), alone, could have an effect possibly strong enough to produce results (A), (B), and (C) above. This can be seen with the use of Fig. 2, which shows the q vs M' relation obtained from (1) and (2), for $\theta = 4.8°$ and for various values of the incident kinetic energy T_0.

It is seen that fortuitously, at the angle used, the value of q for the isobar M_1' (1.23 Bev) was about the same over the energy range covered—namely, slightly under 250 Mev/c. For the isobar M_2' (1.52 Bev) however, even at $T_0 = 2.04$ Bev the value of q is close to 400 Mev/c. Since the cross section for isobar production is propor-

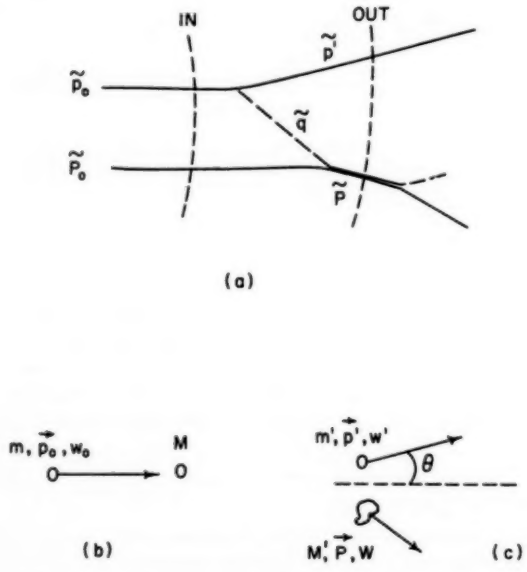

FIG. 1. (a) Representation of isobar production, via a single intermediate pion. The notation used is $\tilde{p}_0 \equiv (\vec{p}_0, iw_0)$; $w_0^2 = p_0^2 + m^2$; etc. (b) Initial state, lab system. (c) "Final" 2-body state, lab system.

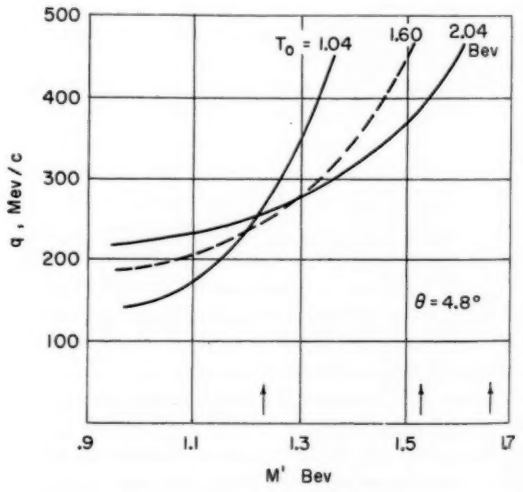

FIG. 2. 4-momentum transfer q vs isobar mass M', conditions of reference (1). The arrows show the expected positions of M_1', M_2', M_3'.

tional to the square of the factors in (3), the net result is that the third factor alone inhibits the production of M_2' by a factor of the order of 5, relative to M_1'.

It seems not unlikely that the form factors in (3) will also be appreciably smaller for $q \sim 400$ Mev/c than for 250 Mev/c, if the effective nucleon size for pion interactions is comparable to that for electromagnetic interactions.

These remarks suggest that for $q \sim 250$ Mev/c (or greater) the isobar excitation probability is decreasing fairly rapidly with increasing q. If this is true it can account for the mass displacement noted as item (B) above, in the following way. If the isobar excitation cross section as a function of M' at constant positive q^2 is peaked at the same value as in the case of real pions ($q^2 = -\mu^2$), and if the peak cross section decreases rapidly with increasing q^2, then data taken along a curve such as the $T_0 = 1.04$ Bev curve in Fig. 2 will yield an apparent displacement of the isobar mass to a lower value. As for the question whether the excitation curve for virtual pions peaks at the same M' as for real pions, it may be noted that in the case of isobar production by virtual photons, this result is in fact true.[3,4] It appears possible that one may be able to argue that the isobar excitation by virtual pions should in this respect show the same feature as for the photon case.[5]

These arguments thus indicate an explanation for items (A) and (B). Item (C), the absence of a low-energy hump for double-M_1' events, also follows. For double-M_1' production at 1.60 Bev, the minimum 4-momentum transfer is $q \sim 500$ Mev/c for $M_1' \sim 1.23$ Bev, or $q \sim 400$ Mev/c for $M_1' \sim 1.18$ Bev. The conclusion is suggested that although appreciable 2-pion production may occur at 1.60 Bev, it does not occur through double-M_1' events.

It seems clear that an analysis of isobar excitation as a function of both q and M' can yield information on the pionic form factors $F_{00}(q^2)$ and $F_{n0}(q^2, M_n')$. Although the elementary approach used here is rather crude, and does not give a reliable method of deriving these form factors from experimental data, it suggests that it would be interesting to try to determine isobar excitation as a function of q and M'. The problem is complicated by the difficulty of distinguishing between "recoil" and "decay" protons.

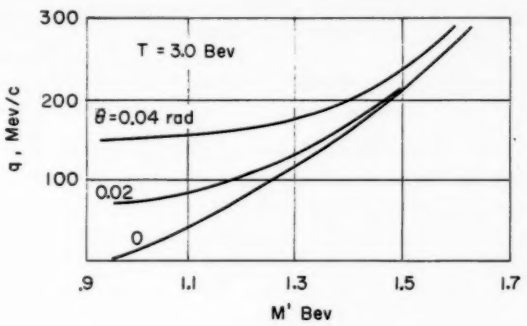

FIG. 3. q vs M', for $T_0 = 3.0$ Bev.

If the arguments given here are correct, then excitation of the higher isobars can be detected only at sufficiently high energies and small angles. Figure 3 shows the q vs M' relation for $T_0 = 3.0$ Bev and a few angles. In so far as one can estimate the spatial extension (and thus in a crude way the dependence of F on q) corresponding to the 1.52-Bev isobar from the magnitude of the corresponding pion-nucleon resonance cross section, it seems possible that even at q as low as 200 to 250 Mev/c, excitation of this isobar may be rather weak. Excitation of this 1.52-Bev isobar may therefore become considerably stronger at energies T_0 appreciably above 3 Bev.

I wish to thank Dr. R. Blankenbecler, Dr. A. Klein, and Dr. H. Primakoff for helpful discussions.

---

*Supported in part by the U. S. Atomic Energy Commission.

[1]G. B. Chadwick et al., Phys. Rev. Letters 4, 611 (1960).

[2]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 105, 1874 (1957).

[3]W. K. H. Panofsky and E. A. Allton, Phys. Rev. 110, 1155 (1958).

[4]S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. 111, 329 (1958).

[5]R. Blankenbecler (private communication). Dr. Blankenbecler has suggested that, drawing on results of dispersion theory for the analogous virtual photon problem, one may expect the form factor $F_{n0}(q^2, M_n')$ to be approximately of the form $F_{00}(q^2)F_{n0}(-\mu^2, M_n')$. In this form, it is evident that the resonance peak, for any fixed q^2, peaks at the same M' as for real pions.

ISOTOPIC SPIN DEPENDENCE OF THE PION-NUCLEON HIGHER RESONANCE

Ronald F. Peierls
Department of Mathematical Physics, University of Birmingham, Birmingham, England
(Received June 17, 1960)

In a recent letter[1] Carruthers has suggested that the "spectrum" of higher resonances in pion-nucleon interactions is different for the $T=1/2$ and $T=3/2$ states; in particular suggesting the existence of a D_{33} resonance in the neighborhood of the third $\pi^- p$ maximum. This seems difficult to reconcile with the experimental results for the branching ratios of the various possible charge states produced by interaction of π^- with protons. The results[2] at 960 Mev are shown in Table I. Their most significant feature is the small total charge exchange scattering. As has been pointed out elsewhere,[3] the ratio of direct elastic to charge-exchange elastic scattering cannot exceed 2, when the elastic $T=3/2$ ($\pi^+ p$) scattering is not negligible, unless there is appreciable interference between the $T=1/2$ and $T=3/2$ amplitudes for $\pi^- p$ scattering, and hence unless the same angular momentum and parity states contribute to both, with approximately the same phase. Since presumably some of the charge-exchange scattering is inelastic, the inequality is quite strongly violated, while if none is inelastic then this is evidence of similar very strong interference terms in the inelastic amplitude. The energy dependence of the $\pi^+ p$ total cross section[4] has a point of inflection at the third $\pi^- p$ maximum (900 Mev), which can be reproduced by adding a small fraction of third $\pi^- p$ peak to a cross section rising steadily towards the fourth maximum at 1.4 Bev. In Carruthers' picture this small extra peak in the $\pi^+ p$ cross section should be due to a D_{33} resonance, and he uses interference between the low-energy portion of this and the D_{13} resonance peaked at 600 Mev to explain the suppression of charge-exchange scattering in the 500-800 Mev region. However, the fact that the charge-exchange scattering is still strongly suppressed, and hence that there is still strong interference, up to 1 Bev is hard to understand on this picture. It seems more reasonable that the interference at this energy should be with the third (possibly $F_{1/2\,5/2}$) resonance, and that the resonant spectrum is the same for both total isotopic spin states.

Table I. Experimental branching ratios in $\pi^- p$ interactions at 960 Mev.

Channel	Cross section (mb)
$\pi^- + p$	19.0
$\pi^0 + n$	7.6
$\pi^0 + \pi^0 + n$	
$\pi^0 + \pi^- + p$	6.8
$\pi^+ + \pi^- + n$	9.5
3π + nucleon	2.7
strange particles	1.3
	(Errors are ~10%)

One possible way of understanding the simultaneous occurrence of the resonances in both isotopic spin states is by attributing their existence[3] to strong interactions in pairs of the particles in final states with two pions. The second and third maxima (600 and 900 Mev) are associated with the interaction of a pion and nucleon through the 33 resonance [Fig. 1(a)]; the fourth maximum (1.4 Bev) is associated with a pion-pion interaction in a state with $t_{\pi\pi}=1$, $j_{\pi\pi}=1$ [Fig. 1(b)]. In each case it is assumed that the charge of the third particle is relatively unimportant and hence that both total isotopic spin states can exhibit the resonance, while the phase of the transition amplitude should not depend on T. At 960 Mev both (a) and (b) should occur and we may expand the state:

$$|\pi\pi N\rangle = a_3 |T=\tfrac{3}{2},\ t_{\pi N}=\tfrac{3}{2}\rangle + a_1 |T=\tfrac{1}{2},\ t_{\pi N}=\tfrac{3}{2}\rangle$$

$$+ b_3 |T=\tfrac{3}{2},\ t_{\pi\pi}=1\rangle + b_1 |T=\tfrac{1}{2},\ t_{\pi\pi}=1\rangle.$$

These four charge states must each be multiplied by an appropriate angular momentum state: one for a_1 and a_3 and a different one for b_1 and

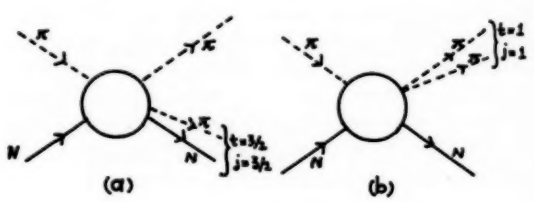

FIG. 1. Mechanisms for the second and third resonances (a) and the fourth resonance (b).

Table II. Theoretical cross sections.

Channel	Relative cross section
$\pi^+ + \pi^- + n$	$\frac{1}{135}[26\|a_3\|^2 + 50\|a_1\|^2 + 14(10)^{1/2}\mathrm{Re}(a_3^* a_1)] + \frac{1}{9}[2\|b_3\|^2 + 2\|b_1\|^2 - 4\mathrm{Re}(b_3^* b_1)]$
$\pi^0 + \pi^- + p$	$\frac{1}{135}[17\|a_3\|^2 + 20\|a_1\|^2 - 10(10)^{1/2}\mathrm{Re}(a_3^* a_1)] + \frac{1}{9}[\|b_3\|^2 + 4\|b_1\|^2 + 4\mathrm{Re}(b_3^* b_1)]$
$\pi^0 + \pi^0 + n$	$\frac{1}{135}[2\|a_3\|^2 + 20\|a_1\|^2 - 4(10)^{1/2}\mathrm{Re}(a_3^* a_1)]$

b_3. The two states [(a) and (b)] are not strictly orthogonal, but the two configurations are so different that their overlap should be small and we neglect their interference in the total cross sections, which are given in Table II.

Experimentally, the distributions for the $n\pi^+\pi^-$ events are well described by the conventional isobar model,[5] which corresponds to $b_1 = b_3 = 0$. From the fact that the π^+ seems to be the "recoil" pion (with higher momentum) about 60% of the time, the assumption made above that the phases of a_1 and a_3 are the same leads to the result that $a_3 \cong \frac{1}{4}a$. But this would predict a branching ratio of 5:1:1.5 for $(n\pi^+\pi^-):(p\pi^-\pi^0):(n\pi^0\pi^0)$ which is quite unlike the observed ratio (Table I). In addition, the distributions for the $\pi^0\pi^- p$ events seem to show evidence of a pion-pion correlation.[6] These facts can all be reconciled if we assume $b_1 \cong b_3 \cong 0.7a_1$, contributing only to $\pi^0\pi^- p$.

There have been several other discussions of possible effects of pion-pion interactions in these processes.[6-8] However, they have mainly been concerned with the absorption mechanism via interaction with the virtual pions emitted by the nucleon. If it is assumed that the interaction pions are emitted directly[6] (Fig. 2), then $b_1 = -\sqrt{2}b_3$, which would predict pion-pion correlations primarily for $n\pi^+\pi^-$ events, contradictory to experiment. The other discussions[7,8] allowing pion-nucleon interactions to take place after the absorption via a pion-pion interaction are particular cases of isobar models of the type (a), and are relevant to the explanation of the relative magnitudes of a_1 and a_3 (or analogous amplitudes in the various models considered) but not to the explanation of final-state pion-pion correlations.

The data at present seem consistent with a picture in which the final states are dominated by the two types of final-state interaction (a) and (b), and in which the phase of the amplitude is independent of T, but the way in which the final states are generated is not understood.

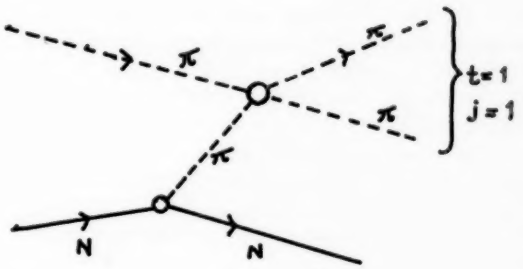

FIG. 2. Direct pion-pion interaction.

I am grateful to Mr. J. H. Gunn for calculating the isotopic spin coefficients, to Mr. D. Radojicic of the Cavendish Laboratory for communicating the experimental results before publication and for useful discussions, and to the Department of Scientific and Industrial Research for the award of a NATO Fellowship.

[1]P. Carruthers, Phys. Rev. Letters 4, 303 (1960).
[2]D. Radojicic (private communication). Similar results are given by V. Alles-Borelli et al., Nuovo cimento 14, 211 (1959); and I. Derado and N. Schmitz, Phys. Rev. 118, 309 (1960).
[3]R. F. Peierls, Phys. Rev. 118, 325 (1960).
[4]T. Devlin et al., Phys. Rev. Letters 4, 242 (1960).
[5]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 109, 1723 (1958).
[6]I. Derado, Nuovo cimento 15, 853 (1960).
[7]L. Landovitz and L. Marshall, Phys. Rev. Letters 4, 474 (1960).
[8]P. Carruthers and H. A. Bethe, Phys. Rev. Letters 4, 536 (1960).

ELECTRON-PROTON SCATTERING AT 900 Mev AND 135°*

L. N. Hand
Physics Department and the High-Energy Physics Laboratory, Stanford University, Stanford, California
(Received July 18, 1960)

Recently a 900-Mev electron beam became available on the linear accelerator at Stanford. This paper reports the results of electron-proton scattering measurements at values of $q^2 > 20$ f^{-2} taken during the adjustment runs of the accelerator. The results, with only statistical errors quoted, are given in Fig. 1 and Tables I and II.

The three points shown on Fig. 1 were obtained with a liquid hydrogen target at $q^2 = 23.9$, 25.1, 26.7 f^{-2} and a scattering angle of 135°. Incident electron energies were 825 Mev, 856 Mev, and 896 Mev. The data were taken with a double-focusing, zero-dispersion spectrometer designed by Alvarez, Brown, Panofsky, and Rockhold,[1] which had installed in it a mask designed to make $\Delta p/p$ independent of angular deviations from the central ray. The nominal $\Delta p/p$ for this experiment was 4%, while $\Delta p/p$ for the incident beam was 1%. The beam was monitored by a Faraday cup known to contain better than 99% of a 600-Mev shower. Detection of the scattered electrons was accomplished by a 10-in. long water Čerenkov counter which makes it possible to reject essentially all the background pulses, while retaining an estimated better than 90% efficiency for real electrons.

We normalized our results with two points at 45°, thus keeping constant the geometry, the recoil kinematic variation over the acceptance aperture, and the momentum of the scattered electron. Only the angle and the incident beam energy were varied. Relative accuracy of the latter was ≤0.1%. The two normalization points were $E_0 = 381$ Mev, $q^2 = 1.95$ f^{-2} and $E_0 = 374$ Mev, $q^2 = 1.88$ f^{-2}. Scattering by the liquid hydrogen caused a possible ≤4% loss in Faraday cup effi-

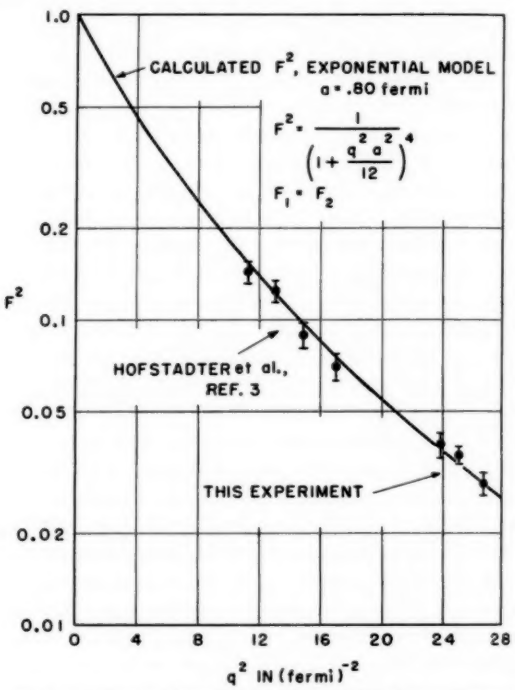

FIG. 1. Ratio of proton elastic scattering to Rosenbluth formula at 135°. (These values of F^2 correspond to those of Table II.)

Table I. Absolute cross sections.

θ	q^2 (f^{-2})	E_0 (Mev)	Physical radiator correction	Schwinger correction	Measured absolute cross sections with corrections (cm^2/sr)	Statistical errors in measured cross sections and in absolute values of F^2	Measured absolute F^2	Calculated[a] F^2
135°	26.7	896	+19%	+31%	0.44×10^{-33}	8%	0.031	0.029
135°	25.1	856	+19%	+31%	0.57×10^{-33}	4%	0.038	0.0335
135°	23.9	825	+19%	+31%	0.61×10^{-33}	9%	0.039	0.0375
45°	1.95	381	+19%	+25%	1.03×10^{-30}	4%	0.72	0.673
45°	1.88	374	+19%	+25%	1.07×10^{-30}	5%	0.72	0.683
45°	1.72	357	+19%	+25%	1.26×10^{-30}	4%	0.78	0.70

[a]Calculated F^2 assumes $F_1 = F_2$ and exponential model.

Table II. Cross-section ratios.

q^2 (f^{-2})		F^2 from measured cross section ratios[a]	Calculated[b] F^2
26.7	$\frac{d\sigma/d\Omega \text{ at } 896 \text{ Mev}, 135^\circ}{d\sigma/d\Omega \text{ at } 381 \text{ Mev}, 45^\circ} = 0.43\times10^{-3}$ (±9%)	0.029 (±9%)	0.029
25.1	$\frac{d\sigma/d\Omega \text{ at } 856 \text{ Mev}, 135^\circ}{d\sigma/d\Omega \text{ at } 374 \text{ Mev}, 45^\circ} = 0.53\times10^{-3}$ (±6%)	0.036 (±6%)	0.0335
23.9	$\frac{d\sigma/d\Omega \text{ at } 825 \text{ Mev}, 135^\circ}{d\sigma/d\Omega \text{ at } 374 \text{ Mev}, 45^\circ} = 0.57\times10^{-3}$ (±10%)	0.039 (±10%)	0.0375

[a] $F(q^2)$ is calculated from exponential model, $F_1 = F_2$ for 45° points.
[b] See footnote (a) of Table I.

ciency at the lower energies, and, of course, we do not know the Faraday cup efficiency at 900 Mev, but more than a 10% shower loss at 900 Mev is not likely.

We calculated absolute cross sections for six points, assuming 100% Faraday cup beam collection and 100% counter efficiency. The absolute normalization was based on the reaction $\gamma + p \rightarrow \pi^+ + n$ with[2] $(d\sigma/d\Omega)_{\gamma-\pi^+} = 16.5$ μb/sr at $k = 250$ Mev and $\theta_{lab} = 83.8^\circ$ plus some current data with the same spectrometer mentioned above taken by R. Alvarez of this laboratory (experiment in progress). The principal limitation is probably in the accuracy to which the absolute π^+ cross section is known. The Schwinger correction was quite large (little affecting the ratios, however), because we are taking a peak height with poor resolution $\langle \ln(E/\Delta E) \rangle_{av} \sim 4$ rather than integrating a curve with good resolution as in the measurements of Hofstadter et al.[3] at lower q^2. The absolute values were taken as a cross-check only.

Probably the principal error in the ratio determination is caused by slight differences in the setting of the spectrometer at the two points, although work at 896 and 856 Mev with high resolution showed that we were within 1/2% of the peak.

Further discussion will have to await more data; however, it appears clear that in the $q^2 = 25$ f^{-2} region, at an angle dominated largely by scattering from the proton's magnetic moment, no structure other than that seen in earlier experiments at lower values of q^2 has appeared.

I wish to thank the accelerator crew for their cooperation in helping to make these measurements possible.

*This work was supported by the joint program of the Office of Naval Research, the U. S. Atomic Energy Commission, and the Air Force Office of Scientific Research.

[1] R. A. Alvarez, K. L. Brown, W. K. H. Panofsky, and C. T. Rockhold, Rev. Sci. Instr. 31, 556 (1960).
[2] A. J. Lazarus, W. K. H. Panofsky, and F. R. Tangherlini, Phys. Rev. 113, 1330 (1959).
[3] R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. 30, 482 (1958); E. E. Chambers and R. Hofstadter, Phys. Rev. 103, 1454 (1956).

EVIDENCE AGAINST THE EXISTENCE OF THE B^0 MESON*

R. Gomez, H. Burkhardt, M. Daybell, H. Ruderman, M. Sands, and R. Talman
California Institute of Technology, Pasadena, California
(Received July 18, 1960)

Recent theoretical work[1-3] on the electromagnetic structure of the nucleon has revived interest in the possibility of a bound state, of 3 pions with $I=0, J=1$. Such a particle might be of assistance in explaining the equality of the isoscalar and isovector parts of the nucleon charge distribution. It should be produced in $\gamma - p$ collisions with a cross section[4] of $\sim 10^{-29}$ cm^2 and would decay rapidly, mainly in the mode

$$B^0 \rightarrow \pi^0 + \gamma. \qquad (1)$$

The lifetime should be around 10^{-20} sec.

This note reports an attempt to observe the production of such a meson in the reaction

$$\gamma + p \rightarrow B^0 + p. \qquad (2)$$

Two previous experiments[5,6] have attempted to measure the cross section for reaction (2) by measuring the proton counting rate at a given proton momentum and angle as a function of the end point of the bremsstrahlung photon beam. No evidence was found for the existence of two-body reactions other than the known π^0 production. The large background due to single π^0 production and π-pair production makes this technique somewhat insensitive.

In the experiment reported here we have used a different technique. Protons at a given angle and momentum were detected in coincidence with a photon on the opposite side of the beam. The photon spectrometer was located at such an angle that the maximum energy of the photons resulting from the decay of singly produced π^0 mesons was lower than the maximum energy of those from the B^0 mesons produced by reaction (2), which decayed according to (1).[7] Those events whose photon energy was between the π^0 cutoff and the B^0 cutoff were attributed to B^0 mesons. This technique is limited to B^0 mesons with mass larger than ~ 3000 Mev; for smaller masses the difference between the two cutoffs is very small. The only possible background is due to π^0-pair production, but the decay photons from this process have only a small probability of falling in the B^0-sensitive region. There is also evidence[8] that the π^0-pair photoproduction cross section is small.

The experimental arrangement is shown in Fig. 1. The 1.1-Bev bremsstrahlung photon beam of the CalTech synchrotron was incident on a liquid hydrogen target. Protons arising from the target were detected by a wedge-shaped magnet and associated counter system previously described by Vette.[9] The decay photons were detected and their energy was measured with a large lead glass Čerenkov counter. When a proton, identified as such by the magnet and counter system, was in coincidence (resolving time 2×10^{-8} sec) with a pulse from the photon detector, a twenty-channel pulse-height analyzer recorded the height of the Čerenkov counter pulse.

The Čerenkov counter was calibrated with momentum-selected electrons and frequently checked by observing the pulse-height distribution from cosmic-ray μ mesons. The resolution of this counter is completely determined by the photoelectron statistics in the photomultipliers. The pulse-height distribution for monoenergetic photons has a standard deviation of

$$\sigma = 6.1(E_\gamma/10^3 \text{ Mev})^{1/2} \text{ Mev}.$$

The acceptance angles for the decay photons, defined by a 4-inch thick lead aperture, were $\pm 8°$ in the direction perpendicular to the plane of production and $\pm 9°$ in the production plane.

Throughout the experiment protons were selected whose momentum was 262 ± 12 Mev/c; the Čerenkov γ-ray detector was located at an angle of 23°, and the end point of the bremsstrahlung photon beam was set at 1130 Mev. Runs were taken at four different proton angle settings, 54°, 57°, 60°, and 66°. The runs at 66° were used to measure the π^0 photoproduction reaction in order to check the over-all efficiency of the system as well as the energy calibration of the Čerenkov counter. Figure 2 shows the observed and calculated Čerenkov counter pulse-height spectrum for the 66° runs. The agreement is quite satisfactory. The cross section for π^0 production obtained from this measurement is 1.26 ± 0.06 μb which is in good agreement with the measurement of Berkelman and Waggoner[10] of 1.1 ± 0.3 μb.

The results of the B^0 runs are presented in Fig. 3 and Table I. The Čerenkov counter pulse-height spectra obtained for the three proton

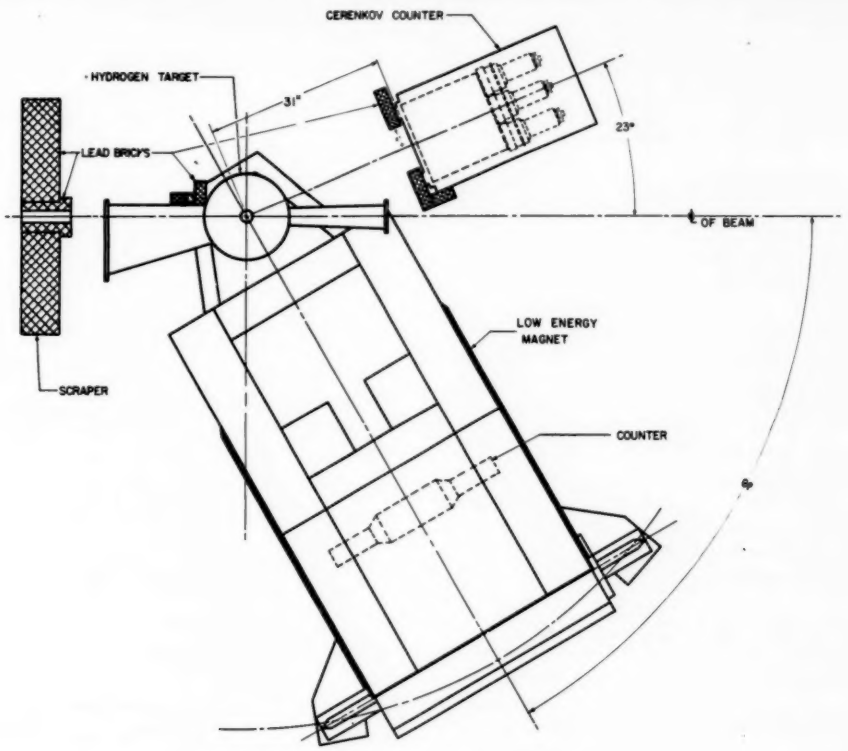

FIG. 1. Experimental arrangement.

spectrometer angles are shown by the broken lines in Fig. 3. The solid line histogram represents the photon spectrum calculated for single π^0 production, with the vertical scale adjusted to give the total of number of counts observed. The smooth solid lines are the spectra expected from B^0 mesons for assumed B^0 masses as indicated in the figure. These spectra were calculated for a differential production cross section of 10^{-30} cm^2 in the center-of-mass system, and the probability of detecting a photon from the π^0 meson which appears in the B^0 decay was neglected. Since the number of counts above the π^0 maximum energy is small, no attempt was made to measure background in this region. The measurements yield, therefore, an upper limit for the cross section of reaction (2). This upper limit, reported in Table I, was obtained by counting those events with energy above a certain energy E_t and calculating the corresponding cross sections for various B^0 masses. E_t was chosen, as indicated by the arrows in Fig. 3, so as to exclude all those

Table I. Average cross section for the photoproduction of B^0 mesons for various B^0 masses. These should be considered as an upper limit, since no backgrounds have been subtracted.

M_B (Mev)	$\sigma_{c.m.}(\theta)$ (cm^2)	Number of counts	$(\theta_B)_{c.m.}$	Incident photon energy interval (Mev)
320	3.2×10^{-32}	5	40°	690 - 1000
350	2.5×10^{-32}	5	38°	750 - 1000
380	2.7×10^{-32}	5	36°	750 - 1050
410	3.4×10^{-32}	4	34°	900 - 1100

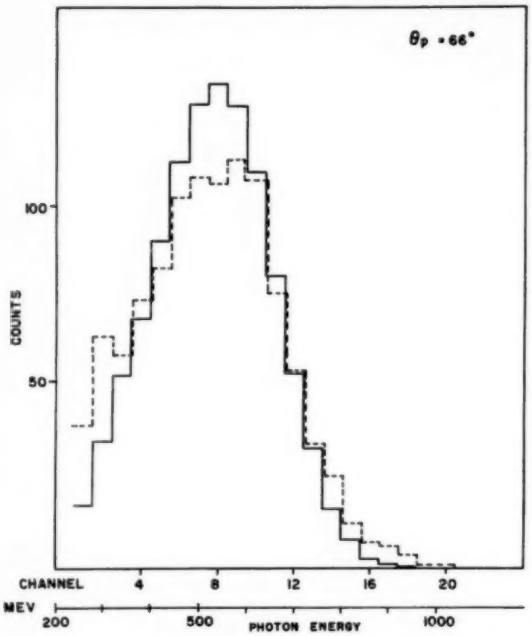

FIG. 2. Photospectra from π^0 meson for $\theta_p = 66°$. The broken line in the measured spectrum. The solid line is the calculated spectrum with the vertical scale adjusted to give the total number of counts observed.

events which clearly correspond to π^0-decay photons. Table I gives the average cross section obtained for four different values of M_B. It also gives the number of events used in calculating these cross sections, the average B^0 center-of-mass angle, and the incident photon energy interval to which these averages correspond.

As indicated above, this experiment becomes insensitive for B^0 masses below 300 Mev. However, if such a particle existed, the following reasoning suggests that it would already have been seen in K^+-decay experiments. Since the B^0 has $I = 0$, the reaction

$$K^+ \rightarrow \pi^+ + B^0 \qquad (3)$$

satisfies the selection rule $\Delta I = 1/2$. Assuming this selection rule is valid, at least to the extent of explaining the factor of 1/500 between the $K_{\pi 2}{}^+$- and $K_{\pi 2}{}^0$-decay rates, the reaction (3) should have a rate based on the $K_{\pi 2}{}^0$-decay rate. If the energy release Q is small, the rate will be reduced by the necessity to generate a p wave and by the smaller phase space. It will be proportional to $Q^{3/2} R^2$, where R is the interaction ra-

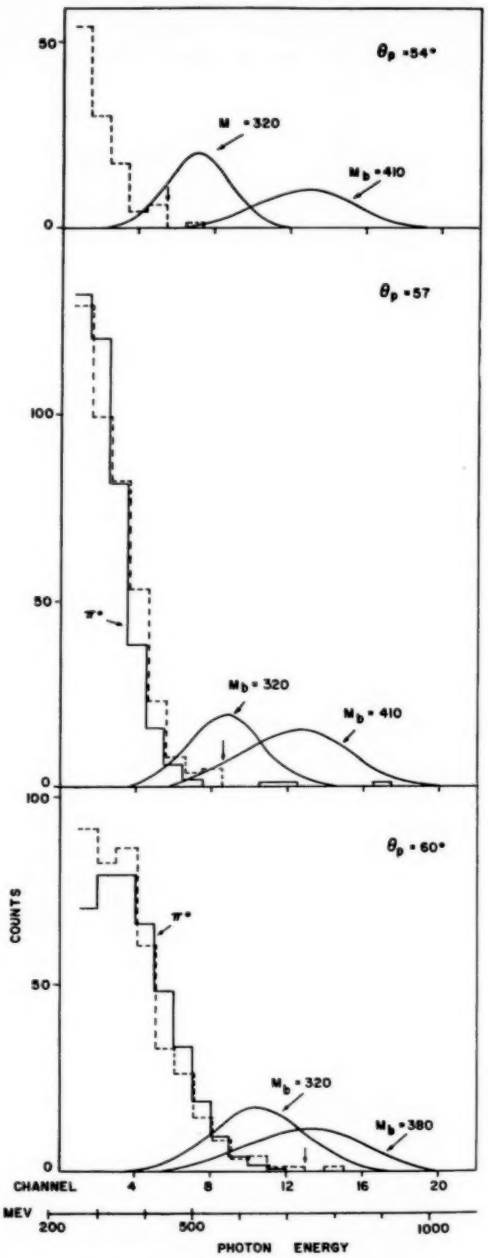

FIG. 3. Photon spectra for various proton angle settings. The broken lines are the observed spectra. The solid line histograms are the calculated spectra from π^0 mesons with the vertical scale adjusted to give the total number of count observed. The solid smooth lines are the expected spectra from B^0 mesons produced with a cross section of 10^{-30} cm^2 in the c.m. system; the various spectra correspond to different values of M_B as indicated in the figure.

dius. Taking the K-meson Compton wavelength for R, we find a rate of 1/250th of the K^0 rate, for a B^0 mass of 320 Mev. The decay mode (3) would appear as a peak in the π^+ energy spectrum from τ' decays. No such peak has been observed;[11] if one assumes that a peak of as much as 10% of the total number of τ' observed might have been missed through lack of statistics, then even this is only 10^{-5} of the K^0 rate, which seems to lie outside the uncertainty of the theoretical estimate. The same argument may also be used to exclude other strongly interacting particles with $J \leq 1$, $I \leq 2$, and $M \lesssim 325$ Mev.

This experiment, incidentally, gives another piece of information, namely a rough lower limit on the B^0 lifetime. Subject to the assumption that (1) is the dominant decay mode and that the cross section is not greatly reduced by accidental cancellations, a limit can be obtained because the decay vertex is also involved in one of the photoproduction matrix elements. At forward angles the nucleon plays a fairly passive role, merely absorbing the low-energy virtual pion, so we can express the probability of the production process in terms of the B^0 lifetime and the pion nucleon coupling constant. We find for the present experiment $\sigma_{c.m.}(\theta) \geq (2.2\times10^{-50}/\tau_B)$; inserting the limits for the cross section obtained in the experiment, we get for the lifetime

$$\tau_B > 10^{-18} \text{ sec.}$$

The authors would like to thank the staff and crew of the CalTech synchrotron for their help through the experiment.

---

*This work was supported in part by the U. S. Atomic Energy Commission.

[1]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959).

[2]G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

[3]A vector meson with these quantum numbers has been discussed by Nambu. See Y. Nambu, Phys. Rev. 106, 1366 (1957).

[4]This number is based on a perturbation theory estimate. If the coupling to the nucleon were much weaker than this it seems unlikely that the particle could account for the larger isoscalar nucleon charge form factor (reference 3).

[5]A. Alberiji et al., reported by G. Bernardini, Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

[6]Matthew Sands (private communication).

[7]Since the experiment is only sensitive to decay mode (1), it can only measure the cross section for (2) times the branching ratio for (1).

[8]The π^0-pair photoproduction cross section seems to be very small for photons from 700 Mev to 1 Bev. See S. Richert and A. Silverman, Bull. Am. Phys. Soc. 5, 237 (1960).

[9]J. I. Vette and W. D. Wales, May 1957 (unpublished report).

[10]K. Berkelman and J. A. Waggoner, Phys. Rev. 117, 1364 (1960).

[11]S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

DISTRIBUTION FUNCTIONS AND QUANTUM STATISTICS. R. U. Ayres, Department of Physics, University of Maryland, College Park, Maryland (Received April 25, 1960).

A new formulation of quantum statistical mechanics is given, in terms of distribution functions. It is shown that all quantities of interest are obtained directly from the distribution of particles in k space, $n(k,\beta)$, and the reaction operator K (often called the Brueckner K matrix) which is familiar from stationary-state many-body perturbation theory. An integral equation for $n(k,\beta)$ is found, which can be solved by a converging iteration process. Some remarks are included on the application of the virial theorem to systems characterized by zero pressure.

d BANDS IN CUBIC LATTICES. II. Joseph Callaway, Division of Physical Sciences, University of California, Riverside, California (Received June 2, 1960).

The results of a previous, perturbation-theoretic treatment of d bands in the body-centered cubic lattice are extended in several respects: The methods of the previous calculation are applied to determine energy levels at the points Γ and X in the Brillouin zone of the face-centered cubic lattice. As before, the crystal potential is that of a lattice of point charges, screened by a uniform distribution of electrons. The perturbation expansion of the wave function of a d electron is developed for the body-centered cubic lattice. Calculations are reported for two states near the top and bottom of the d band, including terms of first order in the potential. These functions have the characteristic property that the wave function of a state near the top of the d band is more compact than that belonging to a state near the bottom. The energies of four states for the body-centered lattice are computed as a function of the binding parameter Za by a more accurate method than that employed in the previous work, making possible an estimation of the accuracy of perturbation theory and the dependence of bandwidth on binding parameter. The role of crystal field effects in the tight-binding limit is discussed, and the circumstances are determined under which the d band may split into sub-bands based on functions of different cubic symmetries. Estimation of the value of the binding parameter for which such separation occurs strongly suggests that this split does not occur for the actual transition metals. Finally, the effects of spin-orbit coupling on the band structure are studied in the tight-binding approximation. A formulation of $\vec{k}\cdot\vec{p}$ perturbation theory for d bands is given.

INFRARED ABSORPTION WAVELENGTHS FOR SOLID LiH AND LiD. W. B. Zimmerman and D. J. Montgomery, Michigan State University, East Lansing, Michigan (Received June 10, 1960).

The infrared absorption spectrum of thin films of LiH and LiD was obtained at room temperature for the region 12.5-25 μ. The primary feature of the spectrum is a broad but definite absorption peak which occurs at 17.0 μ for LiH and at 22.4 μ for LiD. The observed ratio of the wavelengths is 1.32 ± 0.02, in excellent accord with the ratio of the square root of the reduced masses, 1.33. This agreement is a confirmation of the elementary Born theory of lattice vibrations.

SELECTIVE SPIN EXCITATION AND RELAXATION IN NUCLEAR QUADRUPOLE RESONANCE. M. J. Weber* and E. L. Hahn, Department of Physics, University of California, Berkeley, California (Received May 6, 1960).

Nuclear relaxation in a quadrupolar spin system has been investigated by selectively exciting nuclei into particular magnetic levels and observing the transient recovery of the spin system toward an equilibrium population distribution. Selective excitation is achieved by correlating the frequency and precessional behavior of nuclei in certain states with applied elliptically and linearly polarized pulsed radio-frequency fields. A quantum-mechanical analysis is pre-

sented to describe the excitation of a quadrupolar spin system produced by a pulsed elliptically polarized rf field. Using selective excitation techniques, several new modes of longitudinal relaxation are observed. Experiments using the chlorine quadrupole resonance in a single crystal of $KClO_3$ demonstrate how these new relaxation modes are used (1) to study dynamic spin-spin interactions and cross relaxation between overlapping resonance lines, and (2) to determine the individual $\Delta m = \pm 1$ and ± 2 quadrupolar spin-lattice relaxation transition probabilities. A method is introduced by which the magnetic dipole-dipole contribution to the resonance linewidth can be determined independently of static quadrupole broadening, by observing the decay of the beat modulation of certain free-induction signals caused by precession in a small magnetic field. The measured magnetic linewidth of Cl^{35} in $KClO_3$ is in good agreement with the value obtained from a second-moment calculation.

*Present address: Research Division, Raytheon Company, Waltham, Massachusetts.

VALUE OF g' FOR SUPERMALLOY. G. G. Scott, Research Laboratories, General Motors Corporation, Warren, Michigan (Received June 6, 1960).

The gyromagnetic ratio of supermalloy has been determined by measurements of the Einstein-de Haas effect. The value of 1.905 ± 0.002 obtained for g' is equivalent to a spectroscopic splitting factor g of 2.105. This is in good agreement with values obtained by ferromagnetic resonance experiments.

FREE CARRIER ABSORPTION DUE TO POLAR MODES IN THE III-V COMPOUND SEMICONDUCTORS. S. Visvanathan, Research Division, Philco Corporation, Philadelphia, Pennsylvania (Received April 26, 1960; revised manuscript received July 13, 1960).

The longitudinal polar modes of vibration in the III-V compound semiconductors play an important part in determining their transport properties, such as mobility. One would therefore expect them to be important for free carrier absorption as well, in these semiconductors. A quantum mechanical calculation of the free carrier absorption arising from these modes has been made and gives an absorption varying as $\lambda^{2.5}$, and such behavior has been reported experimentally in InP and GaP. The calculated value of the absorption coefficient in InP is in good agreement with experiment.

FREE CARRIER ABSORPTION ARISING FROM IMPURITIES IN SEMICONDUCTORS. S. Visvanathan, Research Division, Philco Corporation, Philadelphia, Pennsylvania (Received April 26, 1960; revised manuscript received July 13, 1960).

The free carrier absorption due to ionized impurities in semiconductors is essentially the inverse process of bremsstrahlung. The cross section for bremsstrahlung is readily available in the literature and one can calculate the spectral distribution of bremsstrahlung for the carriers in a semiconductor; furthermore, by using Kirchhoff's law of radiation, relating the emission and absorption in the semiconductor, one arrives quite easily at the absorption coefficient. The results so obtained agree with those of previous authors who have used a different method of calculation. The inadequacy of the Born approximation in the calculation of the ionized impurity effects on free carrier absorption is brought about clearly in the present treatment.

MAGNETISM AND THE GROUND-STATE ENERGY OF A LINEAR CHAIN. II. CONFIGURATION INTERACTION EFFECTS. David I. Paul, Department of Physics, University of California, Los Angeles, California (Received June 1, 1960).

We continue the investigation, begun in the previous paper of this series, on the ground-state energy of a one-dimensional chain of atoms from the point of view of orthogonal atomic functions. Thus, the exchange integral is positive and the configuration interaction between polar and nonpolar states is included. Our results are extended to include the condition of a large amount of overlap among the unperturbed nonorthogonal atomic functions resulting in a strong interaction between the orthogonalized nonpolar and polar energy states for the case of two electrons having their spins oriented opposite to all other electron spins. We obtain a dispersion relation in accord with the usual anti-

ferromagnetic calculations and thus verify for this case the functional relationship got by using the Dirac spin Hamiltonian with respect to the orthogonality question. However, it is shown that the coefficient is no longer the exchange integral but is instead an interaction integral resulting from the nonorthogonality of the unperturbed atomic functions. Also, the electron spin-spin scattering interaction for the nonpolar configurations appears naturally in our solution for the dispersion relation.

DIRECT EXCHANGE IN FERROMAGNETS. R. Stuart, Lawrence Radiation Laboratory, Livermore, California, and W. Marshall, Atomic Energy Research Establishment, Harwell, England (Received June 6, 1960).

The direct exchange integral which occurs in the Heisenberg theory of ferromagnetism is evaluated for all internuclear spacings. We find that it is always positive; whereas Bethe originally suggested it would be positive only at large spacing and more recently it has been suggested the integral should always be negative. However, at the observed internuclear separation the magnitude calculated is of the order of 70 times too small to explain the experimentally determined exchange constant, and we therefore conclude that direct exchange is not responsible for ferromagnetism in these metals.

THERMAL CONDUCTIVITY OF GERMANIUM FROM 3°K TO 1020°K. Glen A. Slack and C. Glasbrener, General Electric Research Laboratory, Schenectady, New York (Received June 10, 1960).

The thermal conductivity of germanium has been measured from 3°K to 1020°K. From 3°K to 300°K a longitudinal heat flow method was used. A radial flow method employing small samples was used from 300°K to 1020°K. The advantages and special experimental techniques required in this radial method are discussed. From 3°K to 10°K the results are explained by a simple combination of boundary plus isotope scattering in which all phonon-phonon processes can be neglected. At all temperatures below 940°K the heat is carried by phonons, but at 940°K a sharp rise in the thermal conductivity indicates the presence of a second conduction mechanism which may be electron-hole pairs.

THEORY OF THE ETTINGSHAUSEN EFFECT IN SEMICONDUCTORS. B. V. Paranjape and J. S. Levinger, Louisiana State University, Baton Rouge, Louisiana (Received June 1, 1960).

The Ettingshausen effect in semiconductors is mainly due to the generation of electron-hole pairs at one side of the sample and their recombination at the other side. The Ettingshausen coefficient is calculated, in agreement with Putley, as $P=(E_g/\kappa ec)z(1+z)^{-2}(\mu_e+\mu_h)$, where $z=(n_h\mu_h/n_e\mu_e)$ = ratio of hole conductivity to electron conductivity. E_g is the gap energy, and κ the thermal conductivity. We discuss this formula for intrinsic, p-type, and n-type semiconductors. P goes through a maximum for p-type semiconductors near the temperature at which the Hall voltage goes through zero. Our results agree reasonably well with the measurements of Mette, Gärtner, and Loscoe of P as a function of temperature for different samples of germanium and silicon.

KINETICS OF MAGNETIC ANNEALING IN COBALT-SUBSTITUTED MAGNETITE. Wilfred Palmer, International Business Machines Research Laboratory, Poughkeepsie, New York (Received June 3, 1960).

The time dependence of the magnetic annealing effect in single crystals of magnetite containing various amounts of substituted cobalt has been investigated by a technique which permits observation of the effect at the annealing temperature. The annealing kinetics in a particular crystallographic direction are determined by measuring the decay of the torque in the (001) plane following a preparatory anneal in a direction 45° removed. The directions chosen for study are the [100] and [110] directions, which are nodes of the cubic torque curve. The absence of a cubic torque in the directions of measurement allows precise observation of the decay of the anneal-induced uniaxial torque. The annealing kinetics observed in the two directions are different, and the nature of the torque decay in both directions depends upon the cobalt concentration of the sample. The torque decay in the [100] direction is attributed to the redistribution of single cobalt ions and certain pairs of adjacent cobalt ions over the octahedral cation sites, whereas the decay in the [110] direction is attributed to the redistribution of cobalt ion pairs only. The results of a theoretical analysis

of the annealing kinetics that would result from such ionic redistribution are in good agreement with the experimental observations. Comparison of the annealing behavior of samples in different states of oxidation indicates that ionic redistribution occurs by a vacancy diffusion mechanism, and the activation energy observed for this process is 1.05 ± 0.03 ev. In addition to the measurements performed at the annealing temperature, the torque in the [110] direction was determined as a function of temperature in a sample which had been quenched at the end of its preparatory anneal. These two measurements are used to show that there exists a repulsive interaction energy of 0.092 ev between two adjacent cobalt ions.

NUCLEAR RESONANCE ABSORPTION IN Dy^{161} SITUATED IN Dy_2O_3 AND DYSPROSIUM IRON GARNET. S. Ofer, P. Avivi, R. Bauminger, A. Marinov, and S. G. Cohen, Department of Physics, The Hebrew University, Jerusalem, Israel (Received June 1, 1960).

The recoil-free resonant absorption of the 26-kev γ ray ($T_{1/2} \sim 3 \times 10^{-8}$ sec) emitted in the decay of Tb^{161} by absorbers containing Dy^{161} has been investigated. High Mössbauer efficiencies at room temperatures have been observed for sources and absorbers in the form of oxide and rare earth iron garnet. The line shapes obtained were very broad, of the order of 100 times the natural widths, and showed no resolved lines. The broad lines are interpreted as due to a wide complicated hyperfine spectrum whose details have been smoothed out by transitions between magnetic sublevels induced by paramagnetic relaxation. In the rare earth iron garnet, the exchange field acting on the rare earth ion should decouple the nuclear and electron spins. The effective magnetic field at the nucleus in the rare earth garnet is about 2×10^6 oe.

THERMAL EXPANSION OF ALUMINUM AT LOW TEMPERATURES. P. G. Klemens, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received June 15, 1960).

The pressure of the electron gas gives rise to a contribution to the thermal expansion of metals proportional to the dilatational strain derivative of the electronic specific heat. In appears from the strain dependence of the superconducting phase boundary that the electronic specific heat of aluminum is extremely sensitive to dilatation. This would explain why the thermal expansion of aluminum between 20° and 30°K is much larger than expected from the Grüneisen theory. The extreme sensitivity of the electronic specific heat to strain is not inconsistent with the zone structure of aluminum.

THERMALLY ACTIVATED POINT DEFECT MIGRATION IN COPPER. C. J. Meechan, A. Sosin, and J. A. Brinkman, Atomics International, Canoga Park, California (Received April 20, 1960; revised manuscript received June 6, 1960).

A recovery model is presented which includes specific assignments of point defect migration to the various recovery stages in copper. New experimental results showing the effects of prior cold work on the production and subsequent recovery of damage produced in copper by 10°K and 90°K electron irradiations are also presented. The most significant observations in these experiments are: (1) irradiation recovery Stages I_D and I_E are suppressed by previous cold work; (2) this suppression is reflected in an increased damage rate at 90°K; (3) the additional damage which remains in Stages I_D and I_E, or which is produced at 90°K, recovers in Stage III; (4) the recovery in Stage III is altered from the bimolecular process characteristic of annealed copper; (5) under certain conditions a super-recovery occurs in Stage III so that the measured resistivity drops below the pre-irradiation value. These observations are interpreted according to this recovery model.

STUDY OF THE SURFACE STATES OF DIAMOND AND GRAPHITE BY A SIMPLE MO-LCAO METHOD. J. Koutecký and M. Tomášek, Institute of Physical Chemistry, Czechoslovak Academy of Sciences, Prague, Czechoslovakia (Received June 1, 1960).

The surface states of diamond and two-dimensional graphite are investigated on the assumption that the delimitation of the crystal causes no perturbation within the elementary cells of the finite crystal. In both cases a plane perpendicular to the bond between a selected pair of carbon atoms is taken as the delimiting plane. The

molecular orbitals are assumed in the form of a linear combination of sp^3 or sp^2 hybrids. In the case of graphite, molecular orbitals that are linear combinations of $2p_z$ orbitals, whose interaction with sp^2 hybrids is neglected, are further considered. It appears that in the case of diamond there exists a band of energies pertaining to Shockley surface states in the gap between the valence and conductivity bands. The number of atoms in this band equals the number of atoms in the surface. From a discussion of the pertinent wave functions, it follows that these states are an expression of unsaturated bonds of the surface carbon atoms. The electron density on the hybrids projecting from the surface is essentially greater than the density on the other hybrids of the surface atoms. Further bands of surface states exist in the region of energies allowed for the volume valence and the conductivity states. In graphite, a quite analogous behavior is shown by those surface states whose wave functions are linear combinations of sp^2 orbitals. Surface states whose wave functions are linear combinations of p_z orbitals are a manifestation of unsaturated double bonds of surface atoms having only two neighbors. The analogy between these Shockley states and the nonbonding states of odd alternant aromatic hydrocarbons is pointed out.

MODEL OF EXCHANGE-INVERSION MAGNETIZATION. C. Kittel, Department of Physics, University of California, Berkeley, California (Received June 2, 1960).

A thermodynamic theory is given of a class of magnetic crystals which transform from ferromagnetic to antiferromagnetic states as the temperature is varied. Applications are suggested to $Mn_{2-x}Cr_xSb$ and to crystals having the nickel arsenide type structure. It is shown that the exchange magnetoelastic energy is often important in such transformations and leads to an additional interaction energy of the form $(\vec{S}_A \cdot \vec{S}_B)^2$ in the effective spin Hamiltonian. It is suggested that one of the exchange constants goes linearly through zero near a critical value of some lattice coordinate characterizing the transition. Such behavior possibly may result from the combined effects of indirect exchange and superexchange interactions. The Jahn-Teller effect is another possible driving force. There are important differences in the behavior of compact and noncompact antiferromagnetic lattices, under the assumption of nearest-neighbor interactions between sublattices. A triangular array is treated, such as might arise in crystal structures of the NiAs type.

PARAMAGNETIC RESONANCE SPECTRUM OF Mn^{2+} IN $ZnSiF_6{:}6H_2O$, $\Delta m = \pm 1$ TRANSITIONS. E. Friedman and W. Low, Department of Physics, The Hebrew University, Jerusalem, Israel (Received June 13, 1960).

In the spectrum of Mn^{2+} in $ZnSiF_6{:}6H_2O$ a number of weaker lines are observed at intermediate angles of the magnetic field H with respect to the crystal axis, in addition to the 30 allowed transitions $\Delta M = \pm 1$, $\Delta m = 0$. These lines have been measured and assigned to $\Delta M = \pm 1$, $\Delta m = \pm 1$ transitions. The relatively strong intensity is explained and the intensity of the lines is shown to be proportional to $(D/A)^2 \cos^2\theta \sin^2\theta$. These forbidden transitions can be utilized for dynamic polarization of manganese nuclei.

EXCITON AND MAGNETO-OPTICAL EFFECT IN STRAINED AND UNSTRAINED GERMANIUM. David F. Edwards and Vito J. Lazazzera, Willow Run Laboratories, The University of Michigan, Ann Arbor, Michigan (Received June 1, 1960).

Measurements have been made of the direct-transition magneto-optical effect in strained and unstrained germanium at 77°K. The results indicate that the absorption peaks correspond to transitions to exciton levels associated with each Landau level in qualitative agreement with the calculations of Loudon, and Howard and Hasegawa. A definitive experiment is suggested to test this theory.

POLARIZATION OF THE EDGE EMISSION IN CdS. R. J. Collins and J. J. Hopfield, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 2, 1960).

The edge emission spectra of CdS have been examined in the temperature range 70°-150°K. In this interval the ratio of the intensity of the light emitted whose plane of polarization is perpendicular to the c axis of the crystal to that parallel to the c axis was determined. The ratio $I_\perp/I_\parallel$ decreased with increasing temperature. In

agreement with an earlier paper assigning the emission as the recombination of a trapped hole and free electron, an interpretation is offered in terms of the splitting of the ground state of the center. The splitting is caused by the presence of the crystal field in a manner similar to that observed for the valence band. However, from the present experiment alone an equally satisfactory explanation can be offered in terms of the recombination of a trapped electron with holes possessing a thermal distribution among the valence band levels. In either case a splitting of $\Delta E = 0.014$ ev is obtained.

LOW-TEMPERATURE SPECIFIC HEAT OF BODY-CENTERED CUBIC ALLOYS OF 3*d* TRANSITION ELEMENTS. C. H. Cheng, C. T. Wei, and P. A. Beck, Department of Mining and Metallurgical Engineering, University of Illinois, Urbana, Illinois (Received May 20, 1960).

The electronic specific heat coefficient was measured in the temperature range 1.4 to 4.2°K for 48 solid solution alloys in the following binary systems: Ti-V, V-Cr, V-Fe, Cr-Mn, Cr-Fe, and Fe-Co. The electronic specific heat vs electron concentration curves show three quite well separated regions of high density of states. The first of these occurs in alloys with atomic magnetic moments near zero. The second one is found in alloys which have increasing magnetic moments with increasing electron concentration, up to Fe + 35% Co along the Pauling-Slater curve. The third region of high density of states extends from Fe + 35% Co to the limit of the bcc solid solutions at Fe + 75% Co, a range where the magnetic moment decreases with increasing electron concentration.

SPECIFIC HEAT AND ELASTIC CONSTANTS OF SODIUM IODIDE AT LOW TEMPERATURES. Richard N. Claytor and Billy J. Marshall, The Rice Institute, Houston, Texas (Received June 13, 1960).

Measurement of specific heat from 2.4°K to 8°K and of elastic constants from 4.2°K to 300°K have been made on single crystals of sodium iodide. The Debye theta at 0°K, θ_0, as calculated from the specific heat data is 163 ± 1°K. Values of the elastic constants at 4.2°K are $C_{11} = 3.761$, $C_{12} = 0.798$, and $C_{44} = 0.781$ in units of 10^{11} dynes/cm^2. θ_0 calculated from the velocity of sound waves is 167.6 ± 1.5°K. The manner in which the experimental specific heat compares with the work of Berg and Morrison and with the theoretical concepts of Barron and Morrison is discussed.

PARAMAGNETIC BEHAVIOR OF POLYCRYSTALLINE SAMARIUM FROM 300°K to 1400°K. Sigurds Arajs, Edgar C. Bain Laboratory for Fundamental Research, United States Steel Corporation, Research Center, Monroeville, Pennsylvania (Received June 8, 1960).

The paramagnetic susceptibility of polycrystalline samarium metal has been measured as a function of temperature between 300 and 1400°K. The high-temperature phase transformations do not produce noticeable discontinuities in the magnetic susceptibility, indicating that the interactions between the samarium ions are small at these temperatures. The experimental results are compared with the Van Vleck theory of paramagnetism. It appears that the energy levels of Sm^{+++} in metallic samarium differ more from those predicted by the Russell-Saunders coupling than has been realized before. Collective magnetic behaviors resulting from the interactions between the samarium ions are briefly discussed in the light of some recent low-temperature investigations.

THERMOELECTRICITY AND THERMAL CONDUCTIVITY IN THE LEAD SULFIDE GROUP OF SEMICONDUCTORS. Denis Greig, Division of Pure Physics, National Research Council, Ottawa, Canada (Received March 30, 1960; revised manuscript received June 23, 1960).

The thermal conductivity and thermoelectric power of six specimens of PbS and one each of PbSe and PbTe have been measured at temperatures ranging from 4°K to 100°K. In the same temperature region the charge carrier mobility in these samples was determined from measurements of electrical resistivity and Hall coefficient. Four of the PbS samples were natural and *n*-type, while the other specimens were synthetic and *p*-type. The synthetic samples contained only a few single crystals but two of the natural specimens were highly polycrystalline.

At low temperatures the charge carrier mobil-

ities tend to high constant values similar to those reported elsewhere in single crystals of the same materials. From an estimate of the scattering cross section of the point defects we find evidence for explaining this behavior in terms of metallic rather than semiconducting properties.

Maxima attributed to phonon drag have been observed below 20°K in the thermoelectric power of the specimens of highest thermal conductivity. The thermal conductivity was similar in all samples at 100°K but varied by as much as two orders of magnitude at 10°K. In order to explain these results it is necessary to consider scattering of phonons by point imperfections, free electrons, and dislocations.

EFFECTIVE CHARGE OF HEAVY IONS IN VARIOUS MEDIA. P. G. Roll and F. E. Steigert, Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received April 21, 1960).

Effective charge data for heavy ions, as obtained from range-energy curves in oxygen, argon, aluminum, and nickel, are analyzed in terms of a Fermi-Thomas representation of the atom. It would appear that this gives a superior parametrization in terms of nuclear charge. In addition, it appears to properly account for the increase in charge observed for metallic as compared to gaseous absorbers.

RITZ-HYLLERAAS SOLUTIONS OF THE GROUND STATE OF TWO-ELECTRON ATOMS INVOLVING FRACTIONAL POWERS. H. M. Schwartz, Department of Physics, University of Arkansas, Fayetteville, Arkansas (Received December 31, 1959; revised manuscript received July 5, 1960).

Earlier work on the inclusion of half-integral powers in the Ritz-Hylleraas ground-state solutions of the nonrelativistic wave equation for the helium atom is extended through functions involving 18 parameters. Energies that are lower than those found with other published comparable functions, are obtained in all cases. Preliminary results are also given of calculations involving more general fractional powers, Z values different from 2, and half-integral-power solutions for which the expectation value of the square of the Hamiltonian is finite. With the latter type of expansion one obtains, at least at an early stage, an additional improvement in the approximation. Thus with 11 parameters one finds the energy -2.903704 atomic units, which differs by only 0.0007% from the 80-parameter solution of Kinoshita. The initial results found for $Z=8$ indicate an improvement in covergence over that obtained for $Z=2$; the energy -59.156560 atomic units, which was obtained with a 12-parameter function, was only 0.00006% larger than the energy obtained by Pekeris with his 210-term function. The computed mass polarization corrections give also satisfactory results as judged by the similar results obtained with the most extensive solutions available.

HYPERFINE STRUCTURE OF THE METASTABLE $^3P_2$ STATE OF Cd^{111} AND Cd^{113}. W. Faust, M. McDermott, and W. Lichten,* Columbia Radiation Laboratory, Columbia University, New York, New York (Received June 9, 1960).

The hyperfine intervals of Cd^{111} and Cd^{113} $(6s, 6p, {}^3P_2)$ are $\Delta\nu^{111}(F=5/2, F=3/2)=(8232.341 \pm 0.002)$ Mc/sec; $\Delta\nu^{113}(F=5/2, F=3/2)=(8611.586 \pm 0.004)$ Mc/sec. The measured hyperfine anomaly is (0.0016 ± 0.0003)%.

*Present address: Department of Physics, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

BOUNDS ON SCATTERING PHASE SHIFTS: STATIC CENTRAL POTENTIALS. Leonard Rosenberg and Larry Spruch, Physics Department, Washington Square College and Institute of Mathematical Sciences, New York University, New York, New York (Received June 15, 1960).

It has recently been shown that rigorous upper bounds on scattering lengths can be obtained by adding to the Kohn variational expression certain integrals involving approximate wave functions for each of the negative-energy states. For potentials which vanish identically beyond a certain point, it is possible to extend the method to positive-energy scattering; one obtains upper bounds on $(-k\cot\eta)^{-1}$, where η is the phase shift. In addition to the negative-energy states, one must now take into account a finite number of states with positive energies lying below the scattering energy. The states in this associated energy eigenvalue problem are defined by the

imposition of certain boundary conditions on the wave functions. A second approach, involving an associated potential strength eigenvalue problem, is also used. The second method includes the first as a special case and, more significantly, can be extended to scattering by compound systems. If some states are not accounted for, a bound on $\cot\eta$ is not obtained; nevertheless it is still possible to obtain a rigorous lower bound on η. Upper bounds on η may also be obtained, but in a way which is probably not too useful for many-body scattering problems.

FISSION OF RADIUM BY 11- TO 22-Mev PROTONS. Robert L. Wolke,* Department of Chemistry, University of Florida, Gainesville, Florida (Received June 1, 1960).

The ranges and relative yields of Sr^{91} and Pd^{112} fragments from the fission of Ra^{226} by 11- to 22-Mev protons were determined by radiochemical recoil-catching technique. The relative yield data indicate that above 11 Mev the three-humped mass-yield curve becomes predominantly "symmetric" and perhaps somewhat narrower. An apparent fission threshold of ~10.4 Mev was observed. The curve of total fission yield vs energy has the same slope between 13 and 22 Mev as has been reported for the fast-neutron-induced fission of radium. The ranges in mg Au/cm^2 at 11 and 20 Mev, respectively, are, Sr^{91}: 11.2 ± 0.9 and 10.8 ± 0.2, Pd^{112}: 9.1 ± 1.0 and 9.2 ± 0.3.

*Present address: Department of Chemistry, University of Pittsburgh, Pittsburgh, Pennsylvania.

NEUTRON TOTAL CROSS SECTIONS IN THE 17- TO 29-Mev REGION. J. M. Peterson, A. Bratenahl, and J. P. Stoering, Lawrence Radiation Laboratory, University of California, Livermore, California (Received February 18, 1960).

The neutron total cross section of 42 elements and isotopes were measured at several energies between 17 and 29 Mev using monoenergetic neutrons produced by the Livermore variable-energy cyclotron through the $T(d,n)He^4$ reaction. Beam contaminations by gamma rays and low-energy "breakup" neutrons were measured by time-of-flight techniques, and their effects were kept small by suitably high bias on the plastic scintillator detector. The accuracy in cross section is typically ±1 to 2%. The data in this energy region have been sparse heretofore. Where comparisons with previous data have been possible, there is substantial agreement. When plotted versus energy and mass number, the data form a smooth cross-section surface which joins on smoothly to the data at lower energies. The data seem in good agreement with the predictions of the optical model of Bjorklund and Fernbach.

IMAGINARY PART OF THE DELBRÜCK SCATTERING AMPLITUDE. W. Zernik,* Argonne National Laboratory, Argonne, Illinois (Received June 1, 1960).

A method for computing this quantity, based on an expression derived by Kessler, has been developed and applied for gamma-ray energies of 2.62 and 6.14 Mev. The significance of the calculation with regard to available experimental results is discussed and some possible further developments are outlined.

*Now with Westinghouse Atomic Power Division, Pittsburgh, Pennsylvania.

ANGULAR CORRELATION STUDY OF THE $Mg^{24}(d,p\gamma)Mg^{25}$ STRIPPING REACTION AS A TEST OF THE DISTORTED-WAVE THEORY. J. P. Martin,* K. S. Quisenberry, and C. A. Low, Jr., Radiation Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania (Received June 14, 1960).

Angular correlations between protons from the $Mg^{24}(d,p)Mg^{25}$ reaction leading to the 3.40-Mev excited state of Mg^{25} and the resulting de-excitation gamma rays were investigated to test the validity of the distorted-wave stripping theory. A natural magnesium target was bombarded by 15-Mev deuterons and proton-gamma coincidences were counted using scintillation detectors in conjunction with conventional fast-slow coincidence circuitry. The correlations were studied at laboratory proton scattering angles of 15° and 45°, each in two mutually perpendicular planes: the reaction plane and the plane perpendicular to it containing the deuteron axis. The angular distribution of protons from this level was also measured and fitted by a Butler stripping curve with $l_n=1$ and $r_0=5.0$ fermis.

The coordinate system used to describe the correlations is defined with the z axis in the $\vec{k}_d \times \vec{k}_p$ direction and the x axis in the recoil nucleus direction. Correlation functions found by least-squares fits to the experimental data are, for the 15° proton angle,

$$W(\vec{k}_d, \vec{k}_p, \pi/2, \phi) = 1 - (0.385 \pm 0.023) \cos^2(\phi - \phi_0),$$

with $\phi_0 = -27.7° \pm 2.9°$, and

$$W(\vec{k}_d, \vec{k}_p, \theta, \phi_\gamma) = 1 + (0.145 \pm 0.029) \cos^2\theta,$$

where ϕ_γ is the beam direction. The functions found for the 45° proton angle are

$$W(\vec{k}_d, \vec{k}_p, \pi/2, \phi) = 1 - (0.366 \pm 0.033) \cos^2(\phi - \phi_0),$$

with $\phi_0 = -6.8° \pm 3.5°$, and

$$W(\vec{k}_d, \vec{k}_p, \theta, \phi_\gamma) = 1 + (0.279 \pm 0.038) \cos^2\theta.$$

These observed correlations are in good agreement with the predictions of the distorted-wave theory and not with those of the plane-wave theory. It is to be noted in particular that the agreement is excellent at 45° indicating that protons scattered at this angle probably arise from the stripping process in spite of the fact that the disagreement between Butler stripping theory and the measured angular distribution is greatest here. This then suggests that protons observed in the entire region beyond the first maximum of a typical angular distribution are due to stripping and might be adequately described by stripping theory if suitably distorted waves are used in the analysis.

*Now at Brookhaven National Laboratory, Upton, New York.

TIME DEPENDENCE OF RESONANTLY FILTERED GAMMA RAYS FROM Fe^{57}.

F. J. Lynch, R. E. Holland, and M. Hamermesh, Argonne National Laboratory, Argonne, Illinois (Received June 6, 1960).

The time dependence of gamma rays emitted by the 14.4-kev state of Fe^{57} has been studied by delayed-coincidence measurements between a 123-kev gamma ray preceding formation of the state and the 14.4-kev gamma ray from the state. When no filter was used, the number of gamma rays decreased exponentially with the known half-life of 0.1 μsec. When a foil of Fe^{57} (which was resonant to 14.4-kev radiation) was used as a filter, the number of gamma rays observed through the filter did not decrease exponentially. Instead, the filter absorbed almost none of the gamma rays first emitted by the 14.4-kev state; at later times the absorption increased. Data were taken with three different thicknesses of absorber and with emission and absorption peaks separated by 0 to 11 times the width of the resonance. The energy separation resulted from the Doppler shift associated with a constant velocity between source and absorber. These data were, for the most part, in good accord with the prediction of a theory based on a classical model for absorber and source. In particular, the results verified the theoretical prediction that at certain times the intensity of radiation observed would be greater with the filter than without it.

ABSOLUTE MEASUREMENT OF THE $T^3(p,n)He^3$ REACTION THRESHOLD ENERGY AND THE NEUTRON-HYDROGEN MASS DIFFERENCE.

R. O. Bondelid, J. W. Butler, Achilles del Callar,* and C. A. Kennedy, Nucleonics Division, U. S. Naval Research Laboratory, Washington, D. C. (Received June 15, 1960).

The threshold energy of the $T^3(p,n)He^3$ reaction has been precisely measured with a 2-meter-radius electrostatic analyzer, calibrated by absolute methods. A clean tritiated zirconium target, protected from vacuum-system contaminants, was bombarded by protons having energy inhomogeneities as low as 0.02%. The result, obtained by an extrapolation of (net neutron counts)$^{2/3}$, gives a threshold value of 1019.7 ± 0.5 kev, a reaction Q value of 764.3 ± 0.4 kev, and a neutron-hydrogen mass difference of 782.9 ± 0.4 kev.

*Present address: De La Salle College, Bacolod City, Philippine Islands.

ABSOLUTE DETERMINATION OF THE $O^{16}(d,n)F^{17}$ THRESHOLD ENERGY.

R. O. Bondelid, J. W. Butler, and C. A. Kennedy, Nucleonics Division, U. S. Naval Research Laboratory, Washington, D. C. (Received June 15, 1960).

The threshold energy of the $O^{16}(d,n)F^{17}$ reaction has been measured in an absolute manner with a 2-meter-radius electrostatic analyzer. The value, obtained from the intercept of the plot of (net neutron counts)$^{2/3}$ as a function of bombarding energy, is 1829.2 ± 0.6 kev. This

threshold energy gives a reaction Q value of -1.6246 ± 0.0005 Mev, an F^{17} mass of 17.0074990 ± 0.0000032 amu, and a mass excess of 6.9828 ± 0.0030 Mev.

DOUBLE BETA DECAY. II. Lothar Meichsner, Max-Planck-Institut für Chemie, Otto-Hahn-Institut, Mainz, Germany (Received May 25, 1960).

General formulas are given for the differential probability of allowed double β decay of non-oriented nuclei, holding for any initial angular momentum and all possible final angular momenta, according to the theory of Feynman and Gell-Mann. Six different combinations of reduced matrix elements occur. The decay probability for transitions $0 \rightarrow 2$ is small of second order compared with that for transitions $0 \rightarrow 0$. A lower limit for the half-life of the transition $0 \rightarrow 2$ of $_{20}Ca_{28}$ is calculated as a function of the intermediate nuclear energy using j-j shell-model configurations. A slight generalization of the coupling constants suitable for two-neutrino decay is considered.

$Ti^{46,48}(d,p)Ti^{47,49}$ REACTIONS AND THE $1f_{7/2}^{n}$ AND $1f_{7/2}^{n-1}2p$ CONFIGURATIONS. L. H. Th. Rietjens* and O. M. Bilaniuk,† Harrison M. Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan, and M. H. Macfarlane, Physics Division, Argonne National Laboratory, Lemont, Illinois (Received April 25, 1960).

The nuclide Ti^{47} was investigated via a high-resolution spectroscopic study of the reaction $Ti^{46}(d,p)Ti^{47}$. The observed levels, the measured orbital angular momenta of the captured neutrons, and the tentative total angular momenta assigned on the basis of relative intensities were: ground state; 0.16 Mev, $l_n=3$, $J^\pi=\frac{7}{2}^-$; (0.55 Mev, $J^\pi=\frac{3}{2}^-$); 1.56 Mev, $l_n=1$ ($J^\pi=\frac{3}{2}^-$); 1.80 Mev, $l_n=1$ ($J^\pi=\frac{3}{2}^-$); 2.58 Mev, $l_n=1$; 2.83 Mev, $l_n=1$; 3.31 Mev, $l_n=1$ ($J^\pi=\frac{1}{2}^-$ or $\frac{3}{2}^-$); 3.60 Mev ($l_n=1$, $J^\pi=\frac{1}{2}^-$); 3.71 Mev ($l_n=1$, $J^\pi=\frac{1}{2}^-$); and 3.95 Mev ($l_n=1$, $J^\pi=\frac{1}{2}^-$). Similarly for Ti^{49}: ground state, $l_n=3$, $J^\pi=\frac{7}{2}^-$; 1.38 Mev, $l_n=1$, $J^\pi=\frac{3}{2}^-$; 1.72 Mev, $l_n=1$ ($J^\pi=\frac{3}{2}^-$); 2.44 Mev; 2.48 Mev; 3.17 Mev ($l_n=1$, $J^\pi=\frac{1}{2}^-$); and 3.26 Mev, $l_n=1$ ($J^\pi=\frac{1}{2}^-$). It is shown that the ground state of Ti^{49} and the ground state triplet of Ti^{47} can be accounted for by pure $f_{7/2}^{n}$ configurations, and that most of the observed levels can be associated with $1f_{7/2}^{n-1}2p_{3/2}$ and $1f_{7/2}^{n-1}2p_{1/2}$ configurations.

*Now with FOM-Instituut voor Plasma-Fysica, Rijnhuizen, Jutphaas, The Netherlands.

†Now with the Department of Physics and Astronomy, University of Rochester, Rochester, New York.

K-ELECTRON EXCITATION ACCOMPANYING K CAPTURE IN Cs^{131}. N. L. Lark and M. L. Perlman, Chemistry Department, Brookhaven National Laboratory, Upton, New York (Received June 10, 1960).

The probability of production of an atom with a completely vacant K shell by excitation of the second K electron during the K-electron capture process has been determined to be $(2.5 \pm 0.2) \times 10^{-5}$ per K capture event in Cs^{131}. A scintillation coincidence spectrometer with a coincidence resolution time of ten millimicroseconds for 30-kev x rays was used for the measurement. The half-life of Cs^{131} has been redetermined to be 9.69 ± 0.05 days. Previous measurements of electron excitation phenomena accompanying K capture are summarized and the results are compared with the theory of Primakoff and Porter.

RADIATIVE CAPTURE OF PROTONS BY F^{19} AT A BOMBARDING ENERGY OF 669 kev. J. V. Kane, R. E. Pixley, and D. H. Wilkinson,* Brookhaven National Laboratory, Upton, New York (Received June 13, 1960).

The ground-state transition from the 13.51-Mev state of Ne^{20} formed by bombarding fluorine with protons of 669 kev is anomalously weak. This is interesting because the 13.51-Mev state has a large reduced width for proton emission to the ground state of F^{19} and a large $M1$ width for radiation to the first excited (1.63 Mev) state of Ne^{20}. A simple explanation would be that the ground state of Ne^{20} is close to $(2s_{1/2})^4$. The ground-state radiation is sought and found using a 3 in. $\times$ 3 in. NaI(Tl) crystal. The radiative width is approximately 1.0×10^{-2} ev corresponding to $|M|^2 \sim 2 \times 10^{-4}$. The 13.51-Mev state is found also to radiate to the 4.97-Mev state with a width of approximately 0.24 ev. The 4.97-Mev state itself chiefly cascades via the first excited

state: An upper limit on the relative strength of the ground-state transition is 0.07.

*Permanent address: Clarendon Laboratory, Oxford, England.

NUCLEAR ENERGY LEVELS OF N^{17}, O^{18}, AND O^{20}. Nelson Jarmie and M. G. Silbert, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received June 1, 1960).

The reactions $O^{18}(t,\alpha)N^{17}$, $O^{18}(t,p)O^{18}$, and $O^{18}(t,p)O^{20}$ have been studied with a 180°, double-focussing magnetic spectrometer. The measured energy levels of N^{17} up to 4.3 Mev are at 1.374, 1.851, 1.906, 2.536, 3.132, 3.212, 3.652, and 4.010 Mev. The measured energy levels of O^{18} up to 5.2 Mev are at 3.560, 3.639, 3.925, 4.457, and 5.084 Mev. The measured energy levels of O^{20} up to 4.6 Mev are at 1.682, 4.091, and 4.449 Mev. Standard deviations of 15 to 25 kev have been assigned to these excitation energies. The Q value for $O^{18}(t,\alpha)N^{17}$ was determined to be 3.872 ± 0.015 Mev, which leads to a mass for N^{17} of 17.013856 ± 0.000017 amu. The Q value for $O^{18}(t,p)O^{20}$ was determined to be 3.086 ± 0.015 Mev, which leads to a mass for O^{20} of 20.010430 ± 0.000017 amu. Reaction cross sections at 30° lab angle and 2.6-Mev triton energy are presented for each level.

RADIATIVE CAPTURE CROSS SECTIONS FOR FAST NEUTRONS. B. C. Diven, J. Terrell, and A. Hemmendinger, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received June 6, 1960).

Neutron capture cross sections have been measured for 28 elements in the neutron energy range 175 to 1000 kev. The method used was detection of capture gamma radiation in a one-meter-diameter liquid scintillator, the capture samples being placed at the center for irradiation by a pulsed neutron beam. Absolute cross sections have been determined by comparison with the known capture-plus-fission cross section of U^{235}; both capture and fission events are detected with nearly 100% efficiency in this arrangement. Corrections have been made for loss of capture-gamma-ray energy, by means of pulse-height analyses and comparison between one-meter and 48-cm scintillator results; corrections have also been made for change in path length due to scattering. The results obtained appear to agree resonably well with the predictions of resonance theory, with level densities given by the Fermi gas model together with the effective ground state of Hurwitz and Bethe. The low capture cross sections of magic-number target nuclides appear well correlated with the magic-number effects on the mass.

$Be^9(\alpha,n)C^{12}$ REACTION AND THE PARAMETERS OF THE 7.66-Mev STATE OF C^{12}. F. Ajzenberg-Selove, Haverford College, Haverford, Pennsylvania, and P. H. Stelson, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received June 6, 1960).

The $Be^9(\alpha,n)C^{12}$ reaction has been studied at $\bar{E}_\alpha = 5.6$ and 5.78 Mev. Neutron groups have been observed to the first three states of C^{12}. The ratio of the population of the 4.43- and 7.66-Mev states has been determined to be 8.1 ± 1 at $\bar{E}_\alpha = 5.6$ Mev. This value, together with information from a number of other sources, demonstrates that the 7.66-Mev state has $J^\pi = 0^+$, that its width for alpha emission is approximately the Wigner limit ($\Gamma \sim 8$ ev), and that in $\sim 10^{-3}$% of the cases the state decays to the ground state of C^{12} by pair emission. It is also estimated that the 7.66-Mev state can decay by γ emission via the 2^+, 4.43-Mev state with a probability of 1/5000. This information reinforces the proposal that the 7.66-Mev state has the necessary properties to participate in the buildup of the elements in red giant stars.

EXPERIMENTAL STUDY OF THE MECHANISM OF (p,t) REACTIONS AT 22 Mev. J. B. Ball and C. D. Goodman, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received June 3, 1960).

Energy distributions of tritons from various elements throughout the periodic table show prominent peaks which can be identified with states in the residual nucleus which result from the removal of two neutrons from single-particle states in the target. This is indicative of a double-pickup mechanism for the (p,t) reaction at this energy. The use of this reaction in identifying the position and character of single-particle levels is illustrated. The reaction is also used to determine the mass of Ru^{101}.

PARTICLE CREATION IN ELECTRON-ELECTRON COLLISIONS. F. Calogero and C. Zemach,* Istituto di Fisica dell'Università, Roma, Italia and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italia (Received June 9, 1960).

Pair production in high-energy electron-electron collisions is studied, with special attention being given to pion pair production. A method of calculation is formulated which yields results with reasonable directness in the relativistic limit. The orders of magnitude of counting rates for various experimental settings are ascertained. A complete result is obtained for the case in which two pions emerge with equal energies and opposite momenta.

*On leave from the University of California, Berkeley, California.

SEARCH FOR A SECOND π^0. W. Selove and M. Gettner, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 6, 1960).

The energy distribution of neutrons from the charge exchange reaction $\pi^- + p \rightarrow \pi^0 + n$ has been studied, for π^- mesons stopped in liquid hydrogen, to investigate the possible existence of a second π^0 with mass within a few Mev of the mass of the "ordinary" π^0. No neutron group corresponding to such a second π^0 was seen. The sensitivity of the measurement was such that a second group of relative intensity above 10-20% would have been seen for any second π^0 with a mass in the range between about 3/4 Mev and 2 Mev away from the mass of the ordinary π^0.

The data also give a lower limit to the π^0 lifetime: $\tau > \sim 5 \times 10^{-21}$ sec.

PHOTOPRODUCTION OF NEGATIVE MESONS IN DEUTERIUM. D. H. White, R. M. Schectman, and B. M. Chasan, Cornell University, Ithaca, New York (Received May 19, 1960).

A 24-in. diffusion cloud chamber filled to 14 atmospheres with tritium-free deuterium gas, operating in a magnetic field of 6 kilogauss, has been placed in the 1040-Mev "hardened" bremsstrahlung beam of the Cornell synchrotron. A total of 310 events of the type $\gamma + d \rightarrow p + p + \pi^-$ have been observed and analyzed. Using the "spectator model," a total cross section has been determined for the reaction $\gamma + n \rightarrow p + \pi^-$ from threshold to 1 Bev, in good agreement with the results obtained from the measurement of the negative-to-positive pion ratio together with the measured cross section for the reaction $\gamma + p \rightarrow n + \pi^+$.

The momentum distribution of the low-energy protons in the laboratory system compares very favorably with the internal momentum distribution of the deuteron as calculated from the Hulthén or Gartenhaus wave functions. The validity of the spectator model for photons above 200 Mev is experimentally justified, as is expected in consideration of the assumptions of the impulse approximation.

A study of the forward-backward asymmetry of the spectator protons in the laboratory system indicates that above photon energies of 250 Mev, there is a 15% deviation from isotropy, favoring the forward beam direction, indicating that the spectator proton is being "pulled along" with the net forward momentum of the rest of the system, through an average forward momentum transfer of about 10 Mev/c.

PION PRODUCTION FROM $\pi^- - p$ COLLISIONS IN THE LONG-RANGE INTERACTION MODEL. Freda Salzman and George Salzman,* CERN, Geneva, Switzerland and University of Colorado, Boulder, Colorado (Received June 6, 1960).

Long-range interactions of negative pions with protons resulting in single and double pion production are examined in terms of a meson field-theory model in which it is assumed that there is a pion-pion interaction and that the single virtual pion exchange graphs are dominant in large-impact-parameter collisions. This pion exchange leads, in single pion production, to an "excited state," π^*, which "decays" into two pions, and, in double pion production, also to an "excited state," N^*, which "decays" into a pion and a nucleon. For sufficiently high relative energy of the incident π^- and proton, each of these processes can occur for small values of the invariant square of the virtual pion four-momentum, Δ^2, in which case the virtual pion carries very little transverse three-momentum. For small Δ^2 it is shown that it is reasonable to neglect final-state interactions between the "decay products" of the π^* and those of the N^*. It is found that in the limit $\Delta^2 \rightarrow -\mu^2$, where μ is the pion rest mass, the virtual pion behaves kine-

matically, in the π^* barycentric system, as an incoming pion which scatters elastically with the incident π^-, and also behaves, in the N^* barycentric system, as an incoming pion which scatters elastically with the incident proton. Thus, for small Δ^2 in the physical region the π^* and N^* vertices are defined as the corresponding off-the-mass-shell scattering amplitudes. A ratio of appropriately defined double to single pion production cross sections is obtained which is independent of the details of the assumed pion-pion interaction, and depends only on the relative strengths of N^* and N formation. This ratio is estimated by means of the p-wave static nucleon model, applied in appropriate coordinate systems. For incident 5-Bev/c pions this model leads to double pion production which is important compared to single pion production, both because of the $\frac{3}{2}$-$\frac{3}{2}$ pion-nucleon resonance and because of important phase-space factors. Kinematical considerations similar to those described above suggest that this model may also be the theoretical genesis of the "two fireballs" model proposed for ultrarelativistic nucleon-nucleon collisions.

*Present address: University of Colorado, Boulder, Colorado.

MEASUREMENT OF THE GAMMA-RAY SPECTRUM FROM RADIATIVE MUON DECAY. C. O. Kim, W. J. Kernan,* and C. M. York,† The Enrico Fermi Institute for Nuclear Studies and The Department of Physics, The University of Chicago, Chicago, Illinois (Received June 1, 1960).

The energy spectrum of the gamma rays emitted by inner bremsstrahlung in the muon decay process, $\mu^+ \rightarrow e^+ + \nu + \bar{\nu} + \gamma$, has been measured by means of a large sodium iodide crystal. Only those gamma rays which were time-coincident with the decay electrons were recorded. These decay electrons were detected in a scintillation counter telescope adjoining the crystal counter. The counter geometry permitted detection of events with angles between the electron and gamma ray in the range from 0° to 26°. The muons were obtained by stopping a positive pion beam in a block of material. By measuring the gamma-ray spectra from muons originating first in carbon and then aluminum, a correction could be made for bremsstrahlung externally produced by the decay electrons. The measured spectrum is in agreement with that predicted by the theory with an average ratio of 1.02 ± 0.10.

*Present address: Argonne National Laboratory, Argonne, Illinois.

†On leave of absence at CERN, Geneva, Switzerland.

MOMENTUM IMPARTED TO COMPLEX NUCLEI IN HIGH-ENERGY INTERACTIONS. Norbert T. Porile, Chemistry Department, Brookhaven National Laboratory, Upton, New York (Received May 25, 1960).

The momentum imparted to nuclei in the cascade process has been calculated with the aid of the results of a recent Monte Carlo calculation. Results are presented for 0.46-1.84 Bev protons incident on Ru^{100}, Bi^{209}, and U^{238}. The forward and transverse components of momentum of the residual nucleus exhibit a wide range of possible values and are, on the average, approximately equal. The average forward component of momentum increases linearly with the excitation energy of the residual nucleus. The relation between these two quantities leads to considerably lower values of the average excitation energy associated with experimentally determined values of the forward component of momentum than the relations used previously. The calculated momentum values are in most cases consistent with experimental results.

ELECTROPRODUCTION OF PIONS FROM HYDROGEN AND DEUTERIUM. Gerald G. Ohlsen,* Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California (Received June 6, 1960).

The absolute cross section for direct production of pions in electron-proton and electron-deuteron collisions has been measured by the detection of inelastically scattered electrons. Proton data have been taken throughout the range $q^2 = 2.6$ f^{-2} to $q^2 = 10.75$ f^{-2}, and center-of-mass energy $E = 1100$ Mev to $E = 1300$ Mev. Data analysis has been in terms of a neutron magnetic moment distribution. Comparison with available theory yields a neutron rms magnetic moment of 1 f, but better theoretical calculations may change this value somewhat. No theory for the electroproduction of pions from deuterons exists at present. The deuteron data are presented in terms of absolute cross sections as

well as in terms of a deuteron-proton cross-section ratio.

*Now at the University of Texas, Austin, Texas.

PROPOSAL FOR MEASURING THE π^0 LIFETIME BY π^0 PRODUCTION IN ELECTRON-ELECTRON OR ELECTRON-POSITRON COLLISIONS. F. E. Low, Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received June 9, 1960).

The cross section for production of π^0 mesons by colliding electrons is calculated in the virtual-photon approximation. This cross section is directly proportional to the inverse π^0 lifetime, and the proportionality constant is independent of the strong couplings. For a center-of-mass energy of 300 Mev and a π^0 mean life of 10^{-18} sec the total cross section is about 10^{-33} cm^2.

VACUUM POLARIZATION EFFECTS ON ENERGY LEVELS IN μ-MESONIC ATOMS. Roy Glauber, Harvard University, Cambridge, Massachusetts and William Rarita and Philip Schwed, RIAS, Baltimore, Maryland (Received June 13, 1960).

The shifts of the energy levels of μ-mesonic atoms due to vacuum polarization are calculated for the states $n = l+1$. The first-order perturbation integrals are evaluated both in closed form and by means of series expansions. A more accurate calculation of the energy shifts which takes into account the alteration of the atomic wave functions by the vacuum polarization potential is described in the concluding section.

QUASI-ELASTIC PROTON-PROTON SCATTERING AT 158 Mev. B. Gottschalk and K. Strauch, Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received June 15, 1960).

The cross section in impulse approximation for a quasi-elastic proton-proton scattering reaction with a bound target proton depends on the probability of finding inside the nucleus a proton with a momentum equal to the momentum transfer required by kinematics. This momentum transfer is determined if the incident energy and the energies and angles of the two outgoing partners of a proton pair are measured. Events involving s and p protons can be experimentally distinguished by noting that more energy is required to remove an s proton from the nucleus than a p proton. Thus the momentum eigenfunctions of each state can be explored experimentally by varying the momentum transfer, keeping constant the energy sum of a proton pair. Two methods of doing this to bring out the difference between scattering from s and p protons are discussed. They are (1) examination of the distribution of events as a function of the energy sharing between the two outgoing protons, at fixed scattering angles, and (2) examination of the angular distribution of events having fixed sharing. Born approximation calculations are presented for predicting the regions of maximum sensitivity to differences between s and p protons. These predictions are used in the design of two experiments on the reaction $C^{12}(p, 2p)B^{11}$ whose results are reported. They show good qualitative agreement with the predictions for p protons, namely that the sharing will tend to be unequal if the scattering angle of each proton is near 45° [type (1) experiment] and that the angular distribution of events of equal sharing will have a minimum at 45° [type (2) experiment]. The predicted behavior (opposite to that for p protons) for events involving s protons is not observed, and this fact is discussed in the light of possible inadequacies of the theory or of the experiment.

SEARCH FOR EVIDENCE OF PARITY NONCONSERVATION IN K-He INTERACTIONS. M. M. Block, E. B. Brucker, R. Gessaroli,* T. Kikuchi, and C. M. Meltzer, Duke University, Durham, North Carolina, A. Pevsner, P. Schlein, and R. Strand, The Johns Hopkins University, Baltimore, Maryland, H. O. Cohn, Oak Ridge National Laboratory, Oak Ridge, Tennessee, E. M. Harth and J. Leitner, Syracuse University, Syracuse, New York, and A. Minguzzi-Ranzi, L. Monari, and G. Puppi, Istituto di Fisica, Bologna, Italy (Received June 2, 1960).

Parity conservation in strong strange-particle-producing interactions is not yet experimentally settled. We describe a search for evidence of parity nonconservation in K-He interactions. To search for such evidence we measure the expec-

tation value of the pseudoscalar $\vec{P}_\Lambda \cdot \vec{\sigma}_\Lambda$. This quantity is directly obtained from the decay-pion angular distribution in the Λ rest frame. The observed angular distribution is symmetric. An analysis of 485 Λ^0-producing interactions gives $\alpha\langle \vec{P}_\Lambda \cdot \vec{\sigma}_\Lambda \rangle = +0.04 \pm 0.08$. Thus, we find no evidence for parity nonconservation in K-He[4] reactions.

*On leave of absence from the Istituto di Fisica, Bologna, Italy.

INVARIANCE UNDER ANTIUNITARY OPERATORS. G. Feinberg, Department of Physics, Columbia University, New York, New York (Received June 1, 1960).

It is shown that for transitions between "weakly interacting" states, the transition matrix $\mathcal{T}$ can be expressed in terms of a Hermitean operator $\mathcal{T} + \mathcal{T}^\dagger$, and so invariance of the Hamiltonian under antiunitary operators such as T or TCP implies invariance of transition rates under kinematic transformations, without changing the direction of time.

An application is made to π^0 decay into 2 photons, where it is shown that invariance under TCP alone implies equality in the number of left and right circularly polarized photons, to 1 part in 10^4.

TWO-NUCLEON $\vec{L} \cdot \vec{S}$ POTENTIAL IN PSEUDOSCALAR MESON THEORY. C. K. Iddings and P. M. Platzman, California Institute of Technology, Pasadena, California (Received October 5, 1959; revised manuscript received April 1, 1960).

Nonstatic corrections to the two-nucelon potential of Brueckner and Watson and of Gartenhaus are computed within the framework of the γ_5 theory. These terms appear as spin-orbit corrections of order μ/M to the static potentials.

The S matrix is calculated in second and fourth order for a reduced form of the relativistic theory. The potential is then chosen so as to duplicate this S matrix to the required order in the coupling constant and μ/M. We consider to what extent our reduction of the γ_5 theory changes its character.

The resulting potentials are given in analytic form for no cutoff in momentum space and in numerical form for the Gaussian cutoff employed by Gartenhaus. We give also some additional static corrections to previous potentials. A qualitative comparison is made with the experimental observations in nucleon-nucleon scattering, the fine structure in the splitting of the He[5] nucleus, and the contribution of the nonstatic potential to the magnetic moment of the deuteron.

MEASUREMENT OF THE CURVATURE IN A TWO-DIMENSIONAL UNIVERSE. Eugene P. Wigner, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 1, 1960).

The usual definition of the curvature of space involves concepts, such as the measurement of the metric tensor or parallel displacement, which have no direct physical counterpart. The question of obtaining the curvature of a two-dimensional space by means of measurements which are possible at least in principle, is reviewed and a formula given before is corrected and generalized.

INITIAL VALUE PROBLEM OF THE EINSTEIN-MAXWELL FIELD. Louis Witten, Rias, Baltimore, Maryland (Received March 31, 1960; revised manuscript received July 15, 1960).

On an initial hypersurface, $x^0 = 0$, in the presence of gravitation and source-free electromagnetism one can specify the metric tensor, $g_{\mu\nu}$, and its partial derivatives, $g_{\mu\nu,0}$, as well as the electromagnetic tensor, $f_{\mu\nu}$. These quantities must be specified so that on the initial hypersurface two of Maxwell's equations are satisfied and so that the components $R_\alpha{}^0$ of the Ricci tensor are proportional to the components $T_\alpha{}^0$ of the electromagnetic energy-momentum tensor. It is sometimes possible to specify a different electromagnetic tensor on the initial hypersurface which, together with the old metric and Ricci tensors, will describe a properly set initial value problem such that the geometry in advance of the initial hypersurface is different for the different electromagnetic fields. Thus, the Ricci tensor on the initial hypersurface does not always uniquely describe the geometry off the hypersurface in the Einstein-Maxwell theory. The conditions when this nonuniqueness exists are explicitly derived. An initial value problem could be set by specifying $g_{\mu\nu}$ and $g_{\mu\nu,0}$ on the initial hypersurface and deriving an appropriate $f_{\mu\nu}$; however, $g_{\mu\nu}$ and $g_{\mu\nu,0}$ cannot be arbitrarily specified but are subjected to one rather complicated constraint condition on the hypersurface.